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Points fixes d'une application symplectique homologue à l'identité. (French) Zbl 0555.58013

J. Differ. Geom. 22, 49-79 (1985).

Let (M,σ) be a closed symplectic manifold, and (ϕ_t) a Hamiltonian isotopy of M, i.e. $\dot{\phi}_t = X_t \circ \phi_t$ where X_t is a Hamiltonian vectorfield. V. I. Arnold has conjectured that ϕ_1 has at least as many fixed points as a function on M has critical points. Using a method introduced by *C. C. Conley* and *E. Zehnder* for $M = T^{2n}$ [Invent. Math. 73, 33-49 (1983; Zbl 0516.58017)], we prove the following result: Assume M admits a metric of nonpositive curvature such that: 1) $\sigma(X,Y) =< JX, Y >$, where J is an almost complex structure on M; 2) $(\exp_p^* \sigma)_X(Y, JY) \geq \alpha |Y|^2$, with $\alpha > 0$, for $p \in M$ and $X, Y \in T_pM$. Then ϕ_1 has at least $CL(M) + 1$ fixed points, where $CL(M) = \textit{cuplength}$ of $M = \sup\{\ell : \exists$ a ring R and $\omega_1, ..., \omega_\ell \in \tilde{H}^*(M;R)$ with $\omega_1 \cup ... \cup \omega_\ell \neq 0\}$. If the fixed points are nondegenerate then their number is at least $SB(M) = \sup\{rkH^*(M;F)$, F a field$\}$. In particular this proves the Arnold conjecture for surfaces of genus ≥ 1.

MSC:

37J99 Dynamical aspects of finite-dimensional Hamiltonian and Lagrangian systems

57R70 Critical points and critical submanifolds in differential topology

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symplectic manifold; Hamiltonian isotopy; fixed points; critical points; Arnold conjecture

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