**Schikhof, W. H.**

**Ultrametric calculus. An introduction to $p$-adic analysis.** (English) Zbl 0553.26006

Cambridge Studies in Advanced Mathematics, 4. Cambridge etc.: Cambridge University Press. XI, 306 p. (1984).

In the last ten years about ten books appeared on $p$-adic analysis. The one under review is completely different from all of them. Instead of going more or less far in special directions, it exposes, in the author's words, an 'elementary one variable calculus course'; but in that domain it presents practically everything that is known by now. In this choice of the topic it has a predecessor: *D. N. Lenskoĭ* 's "Functions in non-Archimedean normed fields" (Russian) (1962; Zbl 0293.12105). However, the latter book appeared more than twenty years ago when the theory just started to flourish and it has never been widely accessible, so the present treatise is certainly most welcome.

Here is a summary of the contents: Chapter 1 (Valuations) is an introduction consisting of the basic facts about valuations of fields and ultrametric spaces. Chapter 2 (Calculus) deals first with continuity, differentiability, continuous differentiability (the notion is more restrictive than what the name would suggest), antiderivation and integration; then continuous interpolation is used for defining and investigating the Teichmüller character and the gamma function of Overholtzer and Morita; finally, after establishing basic properties about the behaviour of power series, exponential, logarithm and trigonometric functions are presented. Chapter 3 (Functions on $\mathbb{Z}_p$) starts with Mahler's base and its properties, the Volkenborn integral and the Bernoulli numbers; continues with special functions (gamma, log gamma, and zeta); and ends with van der Put's base and differential equations. Chapter 4 (More General Theory of Functions) takes a closer look at differentiability in a sense of real function theory (functions of the first class of Baire, points at which a differentiable function is $C^1$, Lusin's theorem), discusses homeomorphisms, the surjectivity of isometries, and extensions of functions; then it develops a $C^n$-theory and outlines a theory of monotone functions. A few of the topics treated in the main text (e.g. integration) are further advanced in the appendix, mostly by functional analytic means.

The exposition of the material is very clear. Throughout, the author made the effort to present notions and results in the most elementary approach. Some of the results are published here for the first time (but this does not turn out from the presentation, since most results are given without names and all of them without references). Many instructive examples and remarks are included. There are also quite a number of exercises, some of them used afterwards in the main text, too. Many exercises contain hints of solution.

The book will certainly become a standard reference for this part of $p$-adic analysis.

Reviewer: L.Márki

**MSC:**

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|---|---|
| 26E30 | Non-Archimedean analysis |
| 12J25 | Non-Archimedean valued fields |
| 11Sxx | Algebraic number theory: local and $p$-adic fields |
| 11-01 | Introductory exposition (textbooks, tutorial papers, etc.) pertaining to number theory |
| 12-01 | Introductory exposition (textbooks, tutorial papers, etc.) pertaining to field theory |
| 26-01 | Introductory exposition (textbooks, tutorial papers, etc.) pertaining to real functions |



**Keywords:**

elementary $p$-adic analysis; textbook; ultrametric calculus