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Almost periodic Schrödinger operators: A review. (English) Zbl 0545.34023
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From the author's abstract: We review the recent rigorous literature on the one-dimensional Schrödinger equation. $H = -d^2/dx^2 + V(x)$ with V(x) almost periodic and the discrete (= *tight* binding) analog, i.e., the doubly infinite Jacobi matrix, $h_{ij} = \delta_{i,j+1} + \delta_{i,j-1} + V_i\delta_{i,j}$ with V_n almost periodic on the integers. Two themes dominate. The first is that the gaps in the spectrum tend to be dense, so that the spectrum is a Cantor set. We describe intuitions for this from the point of view of where gaps open, and from the point of view of anomalous long time behavior. We give a theorem of Avron and Simon, Chulaevsky, and Moser that for a generic sequence with $\sum |a_n| < \infty$, the continuum operator with $V(x) = \sum a_n \cos(x/2^n)$ has a Cantor spectrum. The second theme involves unusual spectral types that tend to occur. We describe recurrent absolutely continuous spectrum, and show it occurs in some examples of the type just discussed. We give an intuition for dense point spectrum to occur, and some theorems on the occurrence of point spectrum. We sketch the proof of Avron and Simon, that for the discrete case with $V_n = \lambda\cos(2\pi\alpha n + \theta)$, if $\lambda > 2$ and α is a Liouville number, then for a.e. θ,h has purely singular continuous spectrum.

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References:

[1] Avron, J.; Simon, B., J. functional anal., 43, 1, (1981)
[2] Avron, J.; Simon, B., Phys. rev. lett., 46, 1166, (1981)
[3] Avron, J.; Simon, B., Commun. math. phys., 82, 101, (1982)
[4] Avron, J.; Simon, B., Bull. am. math. soc., 6, 81, (1982)
[5] {\scJ. Avron and B. Simon}, Caltech preprint.
[6] {\scJ. Bellisard and D. Testard}, Marseille Preprint, 1980.
[7] {\scJ. Bellisard and D. Testard}, in prep.
[8] {\scJ. Bellisard, A. Formoso, R. Lima, and D. Testard}, Marseille, Preprint, 1981.
[9] {\scV. Chulaevsky}, \textit{Usp. Math}, in press.
[10] {\scR. Johnson}, U.S.C. Preprints, 1980.
[11] Moser, J., Comm. math. helv., 56, 198, (1981)
[12] Johnson, R.; Moser, J., Commun. math. phys., 84, 403, (1982)
[13] Sarnak, P., Commun. math. phys., 84, 377, (1982)
[14] Az'bel, M., Sov. phys. JETP, Phys. rev. lett., 43, 1954, (1979)
[15] Aubry, S., Solid state sci., 8, 264, (1978)
[16] Andre, G.; Aubry, S., Ann. Israel phys. soc., 3, 133, (1980)
[17] Hofstader, D.R., Phys. rev. B, 14, 2239, (1976)
[18] Sokolov, J., Phys. rev. B, 23, 6422, (1981)
[19] Gordon, A.Ya., Usp. math. naut., 31, 257, (1976)
[20] Dinaburg, E.I.; Sinai, Ya., Func. anal. i. pril, 9, 4, 8, (1975)
[21] Russman, ()

[22] Benderskii, M.; Pasteur, L., Math. USSR sb., 11, 245, (1970)

[23] Shubin, M., Trudy, sem. Petrovskii, 3, 243, (1978)

[24] Pasteur, L., Usp. mat. nauk., 28, 3, (1973)

[25] Pasteur, L., Commun. math. phys., 75, 179, (1980)

[26] Kunz, H.; Soulliard, B., Commun. math. phys., 78, 201, (1980)

[27] {\scW. Kirsch and F. Martinelli}, Bielefeld Preprint, 1981.

[28] Goldshade, Ya.; Molchanov, S.; Pasteur, L., Func. anal. i. pril., 11, 1, (1977)

[29] McKean, H.; von Moerbeke, P., Invent. math., 30, 217, (1975)

[30] Dobrovin, D.; Matveev, V.; Novikov, S., Russian math surveys, 31, 59, (1976)

[31] Simon, B., Ann. inst. H. poincare, A 24, 91, (1976)

[32] Reed, M.; Simon, B., Methods of modern mathematical physics, IV. analysis of operators, (1978), Academic Press Princeton, N. J. · Zbl 0401.47001

[33] Eastham, M.S.P., The spectral theory of periodic differential equations, (1973), Scottish Academic Press New York · Zbl 0287.34016

[34] {\scA. Yariv}, private communication.

[35] McKean, H.; Trubowitz, E., Comm. pure appl. math., 29, 143, (1976)

[36] {\scJ. Moser}, private communication.

[37] {\scR. Johnson}, private communication.

[38] Coburn, L.; Moyer, R.; Singer, I., Acta math., 130, 279, (1973)

[39] Connes, A., Advances in math., 39, 31, (1981)

[40] Pimsner, M.; Voiculescu, D., J. operations theory, 4, 201, (1980)

[41] Sacker, R.; Sell, G., J. diff. equations, 27, 320, (1978)

[42] Berezanski, J.M., Expansions in eigenfunctions of self-adjoint operators, (1968), Amer. Math. Soc.

[43] {\scB. Simon}, \textit{Bull. Am. Math. Soc.}, in press.

[44] Schnol, I., Math. sb., 42, 273, (1957)

[45] Simon, B., J. functional anal., 42, 347, (1981)

[46] Pearson, D., Commun. math. phys., 60, 13, (1978)

[47] Choquet, G., ()

[48] Thouless, D., J. phys. C, 5, 77, (1972)

[49] Osceledec, W., Trudy mosk. mat. obsc., 19, 679, (1968)

[50] Ruelle, D., Publ. I.H.E.S., 50, 275, (1979)

[51] {\scJ. Avron and B. Simon}, unpubl.

[52] {\scP. Deift}, private communication.

[53] {\scM. Kac}, public commun. at 1981 AMS Annual Meeting.

[54] Thomas, L., Commun. math. phys., 33, 335, (1973)

[55] {\scJ. Bellisard, R. Lima, and P. Testard}, \textit{Commun. Math. Phys.}, in press.