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Optimality conditions for the bilevel programming problem. (English) Zbl 0537.90087
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The bilevel programming problem (BLPP) is a sequence of two optimization problems where the constraint region of the first is determined implicitly by the solution to the second. Consider two decision-makers or competitive players who must find vectors x and y , respectively, to optimize their individual objective functions $F(x,y)$ and $f(x,y)$. It will be assumed that player 1 has the first choice and selects $x \in X$, followed by player 2 who selects $y \in Y$, where X and Y are nonempty subsets of \mathbb{R}^{n_1} and \mathbb{R}^{n_2} . In addition, the choice made by player 1 may affect the set of feasible strategies, S , open to player 2, implying the existence of jointly dependent constraints.

Letting $S = \{(x, y) : g(x,y) \geq 0\}$ the above situation can be compactly stated as follows:

$$\max_{x \in X} F(x, y), \quad \text{where } y \text{ solves } \max_{y \in Y} f(x, y), \quad \text{subject to } G(x, y) \geq 0.$$

It is first shown that the linear BLPP is equivalent to maximizing a linear function over a feasible region comprised of connected faces and edges of the original polyhedral constraint set. The solution is shown to occur at a vertex of that set. Next, under assumptions of differentiability, first-order necessary optimality conditions are developed for the more general BLPP, and a potentially equivalent mathematical program is formulated. Finally, the relationship between the solution to this problem and Pareto optimality is discussed and a number of examples given.

MSC:

- [90C31](#) Sensitivity, stability, parametric optimization
- [49K35](#) Optimality conditions for minimax problems
- [91A05](#) 2-person games
- [90B50](#) Management decision making, including multiple objectives
- [93A13](#) Hierarchical systems
- [49J35](#) Existence of solutions for minimax problems
- [49K30](#) Optimality conditions for solutions belonging to restricted classes (Lipschitz controls, bang-bang controls, etc.)
- [49J30](#) Existence of optimal solutions belonging to restricted classes (Lipschitz controls, bang-bang controls, etc.)

Cited in **2** Reviews
Cited in **71** Documents

Keywords:

bilevel programming; competitive players; jointly dependent constraints; first-order necessary optimality conditions; Pareto optimality

Full Text: [DOI](#)

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