Moser, Jürgen; Pöschel, Jürgen

An extension of a result by Dinaburg and Sinai on quasi-periodic potentials. (English)

Zbl 0533.34023
Comment. Math. Helv. 59, 39-85 (1984).

We consider the stationary Schrödinger equation $(*)\quad Ly=-y''+qy=\lambda y$ on the real line, where q is a quasi-periodic potential with basic frequencies $\omega=(\omega_1,...,\omega_d)$. For such potentials the rotation number α (λ) is well defined. The spectral gaps of L are precisely the intervals of constancy of α, and there, $\alpha(\lambda)=(j,\omega)/2$ for some integer vector $j=(j_1,...,j_d)$ ("gap labelling").

We suppose that ω is Diophantine, and that q extends to a real analytic function on its hull. Then the following is proven. If $\mu=(k,\omega)/2$ is sufficiently large and badly approximable by all other resonances $(j,\omega)/2$, $j\neq k$, then the spectral gap $[\alpha,\beta]=\alpha^{-1}(\mu)$ is generically open, and (*) has Floquet solutions $e^{i\mu x}(\chi_1+x\chi_2)$, $e^{i\mu x}\chi_2$ for $\lambda=\alpha,\beta$. If the gap is collapsed, that is, if $\alpha=\beta$, then all solutions are of the form $e^{i\mu x}\chi$. The functions χ are all quasiperiodic with frequencies ω and extend to real analytic functions on their hull.

This complements a result of Dinaburg and Sinai who proved the existence of solutions of the second kind for $\lambda=\alpha^{-1}(\mu)$ in case μ is sufficiently large and badly approximable by all resonances $(j,\omega)/2$. In fact, the points in the absolutely continuous spectrum provided by their theorem are cluster points of the spectral gaps constructed above, and their result can be recovered from our construction by a limiting process.

MSC:

34L99 Ordinary differential operators

81Q15 Perturbation theories for operators and differential equations in quantum theory

Keywords:

Schrödinger equation; Floquet solutions; spectral gaps; quasi-periodic potential; rotation number

Full Text: DOI EuDML