Bombieri, E.; **Vaaler, J.**

On Siegel's lemma. (English) Zbl 0533.10030

Invent. Math. 73, 11-32 (1983); addendum ibid. 75, 377 (1984).

The following problem is fundamental in effecting auxiliary constructions in transcendental number theory and diophantine approximation: one has $M < N$ linear equations $\sum_{n=1}^{N} a_{mn}x_n = 0$; $m = 1, 2, ..., M$ with rational integer coefficients a_{mn} not all zero, to be solved for non-zero N-tuples $x = (x_1, ..., x_N)$ of rational integers with a good upper bound for the $|x_n|$. For more refined applications the a_{mn} may be elements of a number field K, whilst the x_n are restricted to integers of a subfield $k \subseteq K$; if $[K : k] = r$ we then need $N > Mr$.

The present paper solves the above problem once-for-all. The authors find and define a canonical height for the given linear system. They prove the existence of N-Mr linearly independent solutions $x_\ell = (x_{1\ell}, x_{2\ell}, ..., x_{N\ell})$ of N-tuples of integers of k so that the product of the heights of these solutions is economically bounded. The results obtained depend on *J. D. Vaaler*'s cube-slicing inequality [Pac. J. Math. 83, 543-553 (1979; Zbl 0465.52011)] and an adèlic generalization of Minkowski's theorem on successive minima [see *R. B. McFeat*, Diss. Math. 88 (1971; Zbl 0229.10014)] proved here independently. These methods are considerably more sophisticated than in Dirichlet's box principle which was traditionally applied in this context.

This paper will influence the future of transcendental number theory and of diophantine approximation.

Reviewer: A.J.van der Poorten

MSC:

11J99	Diophantine approximation, transcendental number theory
11J81	Transcendence (general theory)
11H50	Minima of forms
11R56	Adèle rings and groups

Keywords:

geometry of numbers over adeles; auxiliary functions in transcendence theory; linear equations; canonical height; cube-slicing inequality

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