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Choosability in graphs. (English) Zbl 0469.05032

Combinatorics, graph theory and computing, Proc. West Coast Conf., Arcata/Calif. 1979, 125-157 (1980).

[For the entire collection see Zbl 0435.00002.]

Let $G=(V,E)$ be a graph. Given a function f on the nodes which assigns a positive integer $f(j)$ to node j, assign $f(j)$ distinct letters to node j for each $j\in V$. G is f-choosables if, no matter what letters are assigned to each vertex, we can always make a choice consisting of one letter from each node, with distinct letters from each adjacent node. Using the constant function $f(j)=k$, the choice $\#G$ is equal to k of G is k-choosable but not $k-1$-chooseable. It is shown that choice $\#G\geq\chi(G)$. In fact, choice $\#G\geq\chi(G)$ is unbounded. As an example, it is shown that if $m=\binom{2k-1}{k}$, then $K_{m,m}$ is not k-choosable (where, of course, $\chi(K_{m,m}=2)$). If we denote by $N(2,k)$ the minimum number of nodes in a graph G such that $\chi(G)=2$ but choice $\#G>k$, Then $2^{k-1}\leq N(2,k)\leq k^2 2^{k+2}$. A characterization of 2-choosable graphs is given. Let $\hat{G}$ denote the graph obtained from G by deletion of all nodes with valence 1. Also, let $\theta_{a,b,c}$, denote the θ graph with arcs of length a, b and c, and let C_k denote the closed circuit of length k. Then G is 2-choosable if, and only if, $\hat{G}=K_1$, C_{2m+2} or $\theta_{2,2,2m}$ for $m\geq 1$. It is shown that the graph choosability problem is a π_2^ρ-complete problem. Also let $R_{m,m}$ be a random bipartite graph with bipartitions of size m and with $\frac{\log m}{\log 6}>121$. If $t=\left\lceil\frac{2\log m}{\log 2}\right\rceil$, then with probability $>1-(t!)^{-2}$ we have $\frac{\log m}{\log 6}<\text{choice}\#R_{m,m}<\frac{3\log m}{\log 6}$. Finally, it is noted that the interest in this problem arose in trying to prove J. Dinitz's problem. Given an $m\times m$ array of m-sets, is it always possible to choose on element from each set, keeping the chosen elements distinct in every row, and distinct in every column. This problem remains unsolved for $m\geq 4$.

Reviewer: J.Dinitz

For a scan of this review see the web version.

MSC:

05C15 Coloring of graphs and hypergraphs

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chromatic number; f-choosable; choice $\#G$; 2-choosable graphs; random bipartite graph