Rassias, Themistocles M.

On the stability of the linear mapping in Banach spaces. (English) Zbl 0398.47040

Proc. Am. Math. Soc. 72, 297-300 (1978).

S. M. Ulam posed the problem: Let E_1, E_2 be two Banach spaces, and let $f : E_1 \to E_2$ be a mapping, that is "approximately linear". Give conditions in order for a linear mapping near an approximately linear mapping to exist. The author has given an answer to Ulam's problem. In fact the following theorem has been stated and proved.

Theorem: Consider E_1, E_2 to be two Banach spaces, and let $f : E_1 \to E_2$ be a mapping such that $f(tx)$ is continuous in t for each fixed x. Assume that there exists $\Theta \geq 0$ and $p \in [0, 1)$ such that

$$\frac{\|f(x+y) - f(x) - f(y)\|}{\|x\|^p + \|y\|^p} \leq \Theta,$$

for any $x, y \in \mathbb{R}$. The there exists a unique linear mapping $T : E_1 \to E_2$ such that $\frac{\|f(x)-T(x)\|}{\|x\|^p} \leq \frac{2\Theta}{2-2^p}$, for any $x \in E_1$.

Reviewer: Th. M. Rassias

For a scan of this review see the web version.

MSC:

47H14	Perturbations of nonlinear operators
47A55	Perturbation theory of linear operators
46B99	Normed linear spaces and Banach spaces; Banach lattices

Keywords:

Approximately Linear Mapping

Full Text: DOI

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