Hirsch, Morris W.

Differential topology. (English) Zbl 0356.57001

Graduate Texts in Mathematics. 33. New York - Heidelberg - Berlin: Springer-Verlag. x, 221 p. DM 36.20; $ 14.80 (1976).

There has long been a need for an introductory text on differential topology, and the reviewer must be one of many who have contemplated writing such a book. The appearance of a book by such an appropriate author as Morris Hirsch promises to fill this gap. The philosophy, well expressed in the introduction, emphasizing the origins and applications of smooth manifolds, augurs well, as does the list of contents which proceeds through the commonly agreed key ideas of the subject.

I am afraid that after this I was somewhat disappointed. The author's breezy style, while making the descriptive passages more readable, has been unfortunately extended to a breezy attitude to matters of exposition and proof, which does not. The inexperienced graduate student will have trouble already on page 9, where the "informal discussion of the familiar space S^n" uses the terminology C^r (meaning r times continuously differentiable) with no reference at all; nor will he find any help on this point in the following pages. A similar piece of clumsiness is that the advertised convention that all manifolds are paracompact is spoilt by the fact that the construction of partitions of unity makes no reference to this convention, the crucial step being covered by the unjustified assertion "there is a locally finite atlas V_α such that $\overline{V}_\alpha$ refines U". On these two points, the student would be better advised to read the text (in the same series) by *M. Golubitsky* and *V. Guillemin* on "Stable mappings and their singularities." [New York etc.: Springer-Verlag (1973; Zbl 0294.58004)] whose first two chapters present a well written though concise introduction to several of the topics covered by Hirsch.

Although there are other infelicities of the same kind, these two are perhaps the worst. Another difficulty is that the order of chapters tends to place the harder sections (e.g. function spaces) at the beginning, and the more easily understood geometrical sections (isotopy, surfaces) at the end. The reviewer would prefer to see the integration of vector fields – certainly a key idea historically – introduced early and used as a unifying concept through tubular neighbourhoods, isotopy lemmas and Morse theory (which might then be extended as far as the h-cobordism theorem: the present book discusses the homological and mentions homotopical results, but stops short of handle decompositions).

Another opportunity missed occurs with structure functors (p. 52): these are used only to compare C^r with C^s structures, and density of immersions in the stable range. If this level of abstraction is to be used at all, it would be little extra effort to deduce proofs of tubular and collar neighbourhood theorems, and not much more to obtain Smale-Hirsch immersion theory and Gromov's extension thereof.

But these are blemishes on an otherwise good introductory book. After the two introductory chapters, we are taken reasonably thoroughly through transversality, vector bundles and tubular neighbourhoods, degrees and the Euler characteristic, Morse theory, cobordism and isotopy; and the book closes by applying these basic ideas to classify compact surfaces up to diffeomorphism (unfortunately there is a slip in the statement of the final result: one needs the surfaces to be both orientable or both nonorientable).

It is an attractive book to read, though the absence of precision will irritate some; and in some ways goes further than one might expect (a nice proof of Sard's theorem, and non-trivial discussion of the real analytic case).

A large number of exercises are included: these tend to give extensions of results in the text, relatively few are straightforward calculations. I did not notice any misprints.

Reviewer: C. T. C. Wall (Liverpool)

For a scan of this review see the web version.

MSC:

57-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to manifolds and cell complexes
57Rxx	Differential topology
34C40	Ordinary differential equations and systems on manifolds
54Cxx	Maps and general types of topological spaces defined by maps
55M25	Degree, winding number
55R25	Sphere bundles and vector bundles in algebraic topology
58A05	Differentiable manifolds, foundations
58E05	Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces
57N05	Topology of the Euclidean 2-space, 2-manifolds (MSC2010)

Full Text: DOI