Bühlmann, Hans

Mathematical methods in risk theory. (English) Zbl 0209.23302

Die Grundlehren der mathematischen Wissenschaften. 172. Berlin-Heidelberg-New York: Springer-Verlag. XII, 210 p. (1970).

This is a text-book on the mathematics of the part of actuarial science that is called risk theory. Its author "attempts to create a synthesis out of a selection ...of modern scientific publications in the field of actuarial mathematics, with the goal of presenting a unified system of thought" in replacement of existing fragmented contributions. He has also added new results of his own. The presentation is genuinely probabilistic throughout. It is on the intermediate mathematical level (no measure theory), and the "elements of probability theory which will be necessary for the subsequent developement are recalled [in Chapter 1] for the reader who is moderately familiar with" this type of mathematics. There are parts of the book, however, which must surely require a greater mastery of mathematics than is acquired through actuarial studies in most countries. – The meat of the theory is presented in five chapters. It starts with an account of the risk process which is quite general and where the usual compound Poisson process appears as a special case. The author then goes on to explain his "Bayesian" philosophy where each risk is regarded as a realisation of a random element from a collective, described by a structure function $U(\cdot)$. (Others have called this the approach of "varying (or fluctuating) basic probabilities", and it has been invented to account for he essential heterogeneity of risk portfolios.) In a subsequent chapter, four principles of premium calculation (the expected value principle, the standard deviation principle, the variance principle, and the principle of zero utility) are introduced, and we get a wholly original formulation of credibility theory, based on the variance principle. The next chapter is devoted to retentions and reserves, and it mainly treats the relative retention problem in individual quota and excess of loss reinsurancec. The final chapter covers operational problems of the risk carrier in four main subchapters. There is an account of the probability of ruin and of the solution of the absolute retention problem when the ruin probability is used as the insurance carrier's stability criterion. One finds a subchapter on the dividend policy as the stability criterion. Finally, there is a subchapter on utility theory. The first half of the latter is devoted to an axiomatic introduction to utility, and the second half covers Borch's fundamental theorem on reciprocal reinsurance treaties, as well as price structures with quadratic utility kernels. By intention, the author has left statistical methods (almost) completely out of account, nor does he include numerical methods. Thus, one finds nothing on the Edgeworth expansion, the Esscher approximation, the normal power expansion, and the like. One misses some of the existing reinsurance theory which seems to fit within the framework of the book, notably certain results due to Borsch and Kahn, to Vajda and to 0hlin. – The author gives one numerical example, some exercises are included in each chapter, and there is an index of key-words.

Reviewer: J.M.Hoem

For a scan of this review see the web version.

MSC:

91-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to game theory, economics, and finance
91B30 Risk theory, insurance (MSC2010)
62P05 Applications of statistics to actuarial sciences and financial mathematics