Borsuk, Karol
Concerning homotopy properties of compacta. (English) Zbl 0159.24603
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It is the purpose of this paper to transfer some concepts of the classical homotopy theory (in particular, the concept of the homotopy equivalence of W. Hurewicz and the concept of the homotopy domination of J. H. C. Whitehead) onto arbitrary compacta X, Y lying in the Hilbert space H. In order to do it, the notion of the fundamental sequence $\underline{f} = \{f_k, X, Y\}$ from X to Y is introduced, where $f_k : H \to H$ is a continuous map, for every $k = 1, 2, \ldots$, and for every neighborhood V of Y there exists a neighborhood U of X such that the restriction $f_k|U$ is homotopic in V to the restriction $f_{k+1}|U$ for almost all k. One defines for fundamental sequences the relation of the homotopy and the operation of the composition. Thus one gets a category for which all compacta lying in H are objects and the fundamental sequences are morphisms. This allows to transfer various concepts of the classical homotopy theory onto arbitrary compacta and in particular to introduce the relations of the fundamental equivalence and of the fundamental domination, which in the case of ANR-sets are the same as the relations of the homotopy equivalence and of the homotopy domination, respectively. Every fundamental sequence $\underline{f} = \{f_k, X, Y\}$ induces (covariantly) a homomorphism of the n-dimensional homology group $H_n(X, \mathcal{A})$ onto the group $H_n(Y, \mathcal{A})$ (where $\mathcal{A}$ is an abelian group), and also a homomorphism of the n-th fundamental groups $\underline{\pi}_n(X, x_0)$, being a modification of the n-th homotopy group $\pi_n(X, x_0)$, a modification appropriated to arbitrary compacta.

Besides a development of the general theory of those notions, the paper contains the proof that two plane continua are fundamentally equivalent if and only if their first Betti numbers coincide.

Reviewer: Karol Borsuk

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55-XX   Algebraic topology

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