

**Friedrichs, K. O.**

**On the laws of relativistic electromagneto-fluid dynamics.** (English) Zbl 0308.76075

*Commun. Pure Appl. Math.* 27, 749-808 (1974).

Having noticed that the restriction to strictly hyperbolic equations (as studied by Petrovskiĭ, Leray and others) is too restrictive for general purposes of mathematical physics, the author develops the notions of symmetric hyperbolic systems [cf. author, *Commun. Pure Appl. Math.* 7, 345–392 (1954; [Zbl 0059.08902](#))] and convexly dependent systems [cf. the author and *P. D. Lax*, *Proc. Natl. Acad. Sci. USA* 68, 1686–1688 (1971; [Zbl 0229.35061](#))], and applies them to the system of equations that govern relativistic electromagneto-fluid dynamics.

A recipe for rewriting the basic equations as systems of symmetric hyperbolic equations of the first order is described. It is applicable if the equations are written as a dependent (overdetermined) system of conservation laws enjoying a certain convexity property. In contrast to what happens in the general theory of hyperbolic equations developed by Leray and Ohya, the “initial class”, i.e., the suitable class to which initial functions must belong, may be of finite order (i.e., the initial functions need not be infinitely differentiable), which physically is more plausible. In the case of relativistic magnetofluid dynamics, the system of equations (resp., the forces) has to be modified (resp., have to be chosen) in order that the solution depends on the data with such a restricted sensitivity of finite order. Two such modifications result either in omitting the electric contribution to the electromagnetic forces or in allowing the fluid energy to depend on the charge density in a suitable way.

In the case of polarized electro-magneto-fluid dynamics, the dependent conservation laws, which automatically satisfy the convexity condition, are associated with a Lagrangian. This allows the deduction of a suitable form of the electromagnetic energy-momentum tensor whose expression is closely related to that of *S. R. de Groot* and *L. G. Suttrop* [see their “Foundations of electrodynamics.” Amsterdam: North-Holland (1972)].

Along the same line, the author draws enlightening conclusions as regards, on the one hand, laws involving angular momenta, and on the other, polarized fields with conductivity, where it is shown that Ohmian losses should be related to the rotational character of the flow and have an effect on the angular momentum.

Reviewer: [G rard A. Maugin \(Paris\)](#)

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#### MSC:

[76Y05](#) Quantum hydrodynamics and relativistic hydrodynamics  
[76W05](#) Magnetohydrodynamics and electrohydrodynamics  
[35L45](#) Initial value problems for first-order hyperbolic systems  
[35L65](#) Hyperbolic conservation laws  
[78A25](#) Electromagnetic theory (general)

Cited in **3** Reviews  
Cited in **28** Documents

**Full Text:** [DOI](#)

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