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Quasiconformal groups and a theorem of Bishop and Jones. (English) Zbl 1079.30017

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This paper is a sequel to a series of papers by the authors (with other co-authors) on conformal dynamics, in particular the theory developed by Patterson, Sullivan and Tukia. A basic result in that theory, whose general version is due to Bishop and Jones, states that the Hausdorff dimension of the conical limit set of a non-elementary Kleinian group acting on the hyperbolic space $\mathbb{H}^n$ can be estimated in terms of the exponent of convergence of the Poincaré series of that action [see *C. Bishop* and *P. Jones*, Acta Math. 179, 1–39 (1997; Zbl 0921.30032)]. In the paper under review, the authors obtain such a result in a general setting where the group action is not necessarily Kleinian, but quasiconformal. They obtain new bounds on the exponent of convergence of planar discrete quasiconformal groups in terms of the associated dilatation and the Hausdorff dimension of its conical limit set. They show that if G is a discrete non-elementary K-quasiconformal group acting on $\mathbb{S}^2$, then its exponent of convergence is uniformly bounded, from above by a function of the dilatation K and of the Hausdorff dimension of their conical limit set. More precisely, they prove the following bound:

$$\dim L_C(G) \leq \delta(G) \leq \frac{2K \dim L_C(G)}{2 + (K-1)\dim L_C(G)}$$

where $\dim L_C(G)$ is the Hausdorff dimension of the conical limit set and $\delta(G)$ is the exponent of convergence. The theorem of Bishop and Jones is then obtained as an asymptotic limit in the dilatation.

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MSC:

30C62 Quasiconformal mappings in the complex plane
30F40 Kleinian groups (aspects of compact Riemann surfaces and uniformization)
30F45 Conformal metrics (hyperbolic, Poincaré, distance functions)

Keywords:

Kleinian group; discrete quasiconformal group; exponent of convergence; Hausdorff dimension; conical limit set

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References:

[1] Anderson, G. D., Vamanamurthy, M. K., and Vuorinen, M. Conformal invariants, quasiconformal maps, and special functions, in Quasiconformal space mappings,Lecture Notes in Math. 1508, 1–19, Springer, Berlin, (1992). · Zbl 0767.30018

[2] Anderson, J. W., Bonfert-Taylor, P., and Taylor, E. C. Convergence groups, Hausdorff dimension, and a theorem of Sullivan and Tukia,Geom. Dedicata 103, 51–67, (2004). · Zbl 1067.30041 · doi:10.1023/B:GEOM.0000013844.35478.e5

[3] Astala, K. Area distortion of quasiconformal mappings,Acta Math. 173, 37–60, (1994). · Zbl 0815.30015 · doi:10.1007/BF02392568

[4] Beardon, A. F.The Geometry of Discrete Groups, Springer-Verlag, New York, (1983). · Zbl 0528.30001

[5] Beardon, A. F. The Poincaré metric of plane domains,J. London Math. Soc. 18, 475–483, (1978). · Zbl 0399.30008 · doi:10.1112/jlms/s2-18.3.475

[6] Bishop, C. J. and Jones, P. W. Hausdorff dimension and Kleinian groups,Acta Math. 179, 1–39, (1997). · Zbl 0921.30032 · doi:10.1007/BF02392718

[7] Bishop, C. J. and Jones, P. W. Wiggly sets and limit sets,Ark. Mat. 36, 201–224, (1997). · Zbl 0939.30031 · doi:10.1007/BF02559967

[8] Bojarski, B. V. Homeomorphic solutions of Beltrami systems,Dokl. Acad. Nauk SSR 102, 661–664, (1955).

[9] Bonfert-Taylor, P., Bridgeman, M., and Taylor, E. C. Distortion of the exponent of convergence in space,Ann. Sci. Fenn. 29, 383–406, (2004). · Zbl 1069.30031

[10] Bonfert-Taylor, P. and Taylor, E. C. Hausdorff dimension and limit sets of quasiconformal groups,Mich. Math. J. 49, 243–257, (2001). · Zbl 0999.30029 · doi:10.1307/mmj/1008719771

[11] Bonfert-Taylor, P. and Taylor, E. C. The exponent of convergence and a theorem of Astala,Indiana Univ. Math. J. 51(3), 607–623, (2002). · Zbl 1046.30007 · doi:10.1512/iumj.2002.51.2249

[12] Bonfert-Taylor, P. and Taylor, E. C. Patterson-Sullivan theory and local analysis of limit sets,Trans. Amer. Math. Soc. 355(2), 787–811, (2003). · Zbl 1013.30024 · doi:10.1090/S0002-9947-02-03134-3

[13] Epstein, D. B. A.World Processing in Groups, Jones and Bartlett Publishers, Boston, (1992).

[14] Falconer, K.Fractal Geometry, John Wiley \& Sons, New York, (1990).

[15] Gehring, F. W. TheL P-integrability of the partial derivatives of a quasiconformal mapping,Acta Math. 130, 265–277, (1973). · Zbl 0258.30021 · doi:10.1007/BF02392268

[16] Gehring, F. W. and Martin, G. J. Discrete quasiconformal groups II, unpublished manuscript. · Zbl 0628.30027

[17] Gehring, F. W. and Osgood, B. Uniform domains and the quasihyperbolic metric,J. Anal. Math. 36, 50–75, (1979). · Zbl 0449.30012 · doi:10.1007/BF02798768

[18] Gehring, F. W. and Väisälä, J. Hausdorff dimension and quasiconformal mappings,J. London Math. Soc. 6(2), 504–512, (1973). · Zbl 0258.30020 · doi:10.1112/jlms/s2-6.3.504

[19] Järvi, P. and Vuorinen, M. Uniformly perfect sets and quasiregular mappings,J. London Math. Soc. 54, 515–529, (1996). · Zbl 0872.30014 · doi:10.1112/jlms/54.3.515

[20] Lehto, O. und Virtannan, K. I.Quasikonforme Abbildungen, Grundlehren der Mathematischen Wissenschaften, Springer-Verlag, (1965).

[21] Sullivan, D. On the orgodic theory at infinity of an arbitrary discrete group of hyperbolic motions, Riemann surfaces and related topics: Proceedings of the 1978 Stony Brook Conference,Ann. of Math. Stud. 97, Princeton University Press, 465–496, (1981).

[22] Tukia, P. On two-dimensional quasiconformal groups,Ann. Acad. Sci. Fenn. Ser. A Math. 5, 73–78, (1980). · Zbl 0411.30038