Imayoshi, Yoichi; Ito, Manabu; Yamamoto, Hiroshi

**A reducibility problem for monodromy of some surface bundles.** (English) Zbl 1080.57021

J. Knot Theory Ramifications 13, No. 5, 597-616 (2004).

Let $X$ denote an orientable closed surface of genus $g$ with $n$ points removed. *E. Fadell* and *L. Neuwirth* in [Math. Scand. 10, 111–118 (1962; Zbl 0136.44104)] defined for each integer $n \geq 0$ a fibration $F_1 X_n \to F_{n+1}X \to F_n X$ where $X_n = X \setminus \{x_1^0, \dots x_n^0\}$ and $F_n$ denotes the $n$th configuration space. This fibration naturally defines a monodromy $\pi_1(F_n X) \to Iso(X,n)$, where $Iso(X,n)$ is the group of isotopy classes of orientation preserving homeomorphisms $f : X_n \to X_n$. This work describes a relation between elements $\beta \in \pi_1(F_n X)$ and their image $f \in Iso(X_n)$ with respect to the Thurston classification of the surface automorphisms. More precisely using the notation above the authors show:

Theorem: Let $X$ be an oriented surface of non-excluded finite type $(g,m)$; that is, $2g-2+m>0$. Then the element $1 \neq f \in Iso(X,n)$ is not of finite order. Further, $f$ is reducible if and only if it can be induced by $\beta \in \pi_1(F_n X)$ satisfying at least one of the following conditions: (i) $\beta$ is non-spreading; (ii) $\beta$ has a boundary partition; (iii) $\beta$ has a tube structure over some subset in $\{1, \dots, n\}$ which is not a singleton.

One crucial step is to show, with the help of Teichmüller theory, that a mapping class $f \in Iso(X,n)$ is reducible if it can be induced by a braid $\beta \in \pi_1(F_n X)$ satisfying none of the conditions (i), (ii) and (iii). The paper is well organized and it provides several intuitive geometrical interpretations.

Reviewer: Daciberg Gonçalves (São Paulo)

**MSC:**

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|---|---|
| 57M99 | General low-dimensional topology |
| 57M50 | General geometric structures on low-dimensional manifolds |
| 55S37 | Classification of mappings in algebraic topology |
| 20F36 | Braid groups; Artin groups |
| 57N05 | Topology of the Euclidean 2-space, 2-manifolds (MSC2010) |



**Keywords:**

surface bundles; monodromy; Thurston's classification of surface automorphisms; Teichmüller space; braid groups; Fadell-Neuwirth sequence

**Full Text:** DOI

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[13] DOI: 10.1142/S0218216595000259 · Zbl 0874.57010 · doi:10.1142/S0218216595000259

paper as accurately as possible without claiming the completeness or perfect precision of the matching.