Lorenz, Falko

Käte Hey and the Main Theorem in the theory of algebras. (Käte Hey und der Hauptsatz der Algebrentheorie.) (German) Zbl 1095.01010

Mitt. Math. Ges. Hamb. 23, No. 2, 75-92 (2004).

Käte Hey (1904–1990) was the first Ph.D. student of Emil Artin. In her dissertation ["Analytische Zahlentheorie in Systemen hyperkomplexer Zahlen". Hamburg, Diss. (1929; JFM 56.0888.02)], she studied the zeta function of division algebras over number fields at a time when the only examples known were essentially the quaternion algebras.

In order to understand Hey's results, let us introduce the relevant notation. Let K denote a global or a local field; a division algebra over K is a division algebra D with center K such that the dimension of D over K is finite. We say that a K-algebra A splits over K if A is isomorphic to the matrix algebra $M_n(K)$ for some integer n. We say that A splits at a prime $\mathfrak{p}$ of K if the algebra $A_{\mathfrak{p}} = A \otimes_K K_{\mathfrak{p}}$ splits over the completion $K_{\mathfrak{p}}$ of K. The main theorem for algebras then states that a division algebra D over some number field K splits over K only if it splits at every prime $\mathfrak{p}$ of K. This local-global principle was proved by Brauer, Hasse and Noether in 1931 [JFM]. Implicitly, this result is one of the main theorems of Hey's dissertation.

In this article, the author presents what he could find out about Hey's biography, discusses the content of Hey's thesis, mentions a few gaps later closed by Zorn, and finally studies connections with Hasse's sum formula in class field theory.

Reviewer: Franz Lemmermeyer (Bilkent)

MSC:

01A60 History of mathematics in the 20th century
01A70 Biographies, obituaries, personalia, bibliographies

Keywords:

Hey; local fields; division algebras; Brauer group; zeta function; local-global principle; analytic number theory

Biographic references:

Hey, Käte