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Prescribing analytic singularities for solutions of a class of vector fields on the torus. (English)

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This paper deals with the analytic singularities of the operator

$$L = \partial_t + (a(t) + ib(t))\partial_x$$

acting on distributions on the torus $\mathbb{T}_{t,x}^2$. The functions a, b are real valued, real analytic on \mathbb{T}_t^1 . We shall formulate some of the main results of the paper. Assume that b changes sign and Σ is any subset of the set of the local extrema of the local primitives of b . Then there exists a solution $u \in D'(\mathbb{T}^2)$ of $Lu = f \in C^\omega(\mathbb{T}^2)$ such that the t -projection of its analytic singular support is Σ . Moreover, for any $\tau \in \Sigma$ and any closed $F \subset \mathbb{T}_x^1$ one can find $u \in D'(\mathbb{T}^2)$ such that $Lu \in C^\omega(\mathbb{T}^2)$ and $\text{sing supp}_A(u) = \{\tau\} \times F$. The results here proposed are sharp, i.e. if t is neither a local minimum nor a local maximum, then every $u \in D'(\mathbb{T}^2)$, for which $Lu \in C^\omega(\mathbb{T}^2)$, is real analytic in (t, x) .

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