**Slepyan, L. I.**; **Ayzenberg-Stepanenko, M. V.**
**Localized transition waves in bistable-bond lattices.** (English) Zbl 1159.74390
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Summary: Discrete two-dimensional square- and triangular-cell lattices consisting of point particles connected by bistable bonds are considered. The bonds follow a trimeric piecewise linear force-elongation diagram. Initially, Hooke's law is valid as the first branch of the diagram; then, when the elongation reaches the critical value, the tensile force drops to the other. The latter branch can be parallel with the former (mathematically this case is simpler) or have a different inclination. For a prestressed lattice the dynamic transition is found analytically as a wave localized between two neighboring lines of the lattice particles. The transition wave itself and dissipation waves carrying energy away from the transition front are described. The conditions are determined which allow the transition wave to exist. The transition wave speed as a function of the prestress is found. It is also found that, for the case of the transition leading to an increased tangent modulus of the bond, there exists nondivergent tail waves exponentially localized in a vicinity of the transition line behind the transition front. The previously obtained solutions for crack dynamics in lattices appear now as a partial case corresponding to the second branch having zero resistance. At the same time, the lattice-with-a-moving-crack fundamental solutions are essentially used here in obtaining those for the localized transition waves in the bistable-bond lattices. Steady-state dynamic regimes in infinite elastic and viscoelastic lattices are studied analytically, while numerical simulations are used for the related transient regimes in the square-cell lattice. The numerical simulations confirm the existence of the single-line transition waves and reveal multiple-line waves. The analytical results are compared to the ones obtained for a continuous elastic model and for a related version of one-dimensional Frenkel-Kontorova model.

**MSC:**

74N20 Dynamics of phase boundaries in solids
74J99 Waves in solid mechanics



**Keywords:**

A. Dynamics; B. Phase transition; Bistable-bond lattice; C. Integral transforms

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