Wanless, Ian M.; Ihrig, Edwin C.

Symmetries that Latin squares inherit from 1-factorizations. (English) Zbl 1067.05061

J. Comb. Des. 13, No. 3, 157-172 (2005).

A rooted 1-factorization, (F, v), of the complete graph K_{n+1}, with n odd, consists of a 1-factorization F of K_{n+1} along with a vertex v from K_{n+1}, called the root, and a total order on the vertices of K_{n+1}. The authors make use of the well-known $\mathbb{K}$-construction to obtain an ordered 1-factorization of the complete bipartite graph $K_{n,n}$ and then a Latin square of order n denoted by $\mathcal{L}(F, v)$, which is shown to be symmetric and idempotent, by not necessarily totally symmetric. Conversely, any symmetric idempotent Latin square of order n is proven to be $\mathcal{L}(F, v)$ for some (F, v) of K_{n+1}. They establish a number of results concerning the autotopies which $\mathcal{L}(F, v)$ can possess. In particular, the automorphism group of $\mathcal{L}(F, v)$ is isomorphic to the stabilizer of v in the automorphism group of F. Their main result is that the three assertions: $\mathcal{L}(F, u)$ is paratopic to $\mathcal{L}(G, v)$, $\mathcal{L}(F, u)$ is isomorphic to $\mathcal{L}(G, v)$, and (F, u) is isomorphic to (G, v), are equivalent. They also provide an algorithm that can determine in $O(n^3)$ time whether a Latin square of order n is paratopic to an $\mathcal{L}(F, v)$, or whether it is isotopic to a symmetric or totally symmetric Latin square.

Reviewer: Chester J. Salwach (Easton)

MSC:

05C70 Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)

05B15 Orthogonal arrays, Latin squares, Room squares

05-04 Software, source code, etc. for problems pertaining to combinatorics

Keywords:

perfect 1-factorization; autotopy; paratopy; main class; totally symmetric; idempotent; atomic Latin square

Full Text: DOI

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