Liskovets, Valery A.; **Walsh, Timothy R.**

Counting unrooted loopless planar maps. (English) Zbl 1070.05050

Eur. J. Comb. 26, No. 5, 651-663 (2005).

This paper is devoted to proving the following formula for $L^+(n)$, the number of loopless planar maps with n edges up to an orientation-preserving isomorphism:

Theorem 1. For $n \geq 1$,

$$L^+(n) = \frac{1}{2n}\left[\frac{2(4n+1)}{(n+1)(3n+1)(3n+2)}\binom{4n}{n} + \sum_{t<n,t|n} \phi\left(\frac{n}{t}\right)\binom{4t}{t} + \begin{cases} \frac{2n}{n+1}\binom{2n}{\frac{n-1}{2}} & \text{if } n \text{ is odd} \\ \binom{2n}{\frac{n-2}{2}} & \text{if } n \text{ is even} \end{cases}\right],$$

where $\phi(n)$ is the Euler totient function.

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MSC:

05C30 Enumeration in graph theory

Cited in **1** Review
Cited in **2** Documents

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