Galatos, Nikolaos; **Tsinakis, Constantine**
Generalized MV-algebras. (English) Zbl 1063.06008
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The authors define a generalized MV-algebra (GMV-algebra for short) as a residuated lattice satisfying the identities $x/((x \vee y) \setminus x) = x \vee y = (x/((x \vee y)) \setminus x$. A closure operator γ on a residuated lattice $\mathbf{L}$ such that $\gamma(a)\gamma(b) \leqq \gamma(ab)$ for all $a, b \in L$ is called a nucleus on L; the image L_γ of γ is endowed with a residuated lattice structure $\mathbf{L}_\gamma = (L, \wedge, \vee_\gamma, \circ_\gamma, \setminus, /, \gamma(e))$, where $\gamma(a) \vee_\gamma \gamma(b) = \gamma(a \vee b)$ and $\gamma(a) \circ_\gamma \gamma(b) = \gamma(ab)$.

The fundamental result of the paper is the following theorem: A residuated lattice $\mathbf{M}$ is a GMV-algebra if and only if there are residuated lattices $\mathbf{G}, \mathbf{L}$, such that $\mathbf{G}$ is an ℓ-group, $\mathbf{L}$ is the negative cone of an ℓ-group, γ is a nucleus on $\mathbf{L}$ and $\mathbf{M} = \mathbf{G} \oplus \mathbf{L}_\gamma$ (where $\oplus$ denotes the operation of the direct sum). As a consequence, the authors obtain a categorical equivalence that generalizes the results of Mundici and Dvurečenskij concerning the functor Γ. Further, they prove that the equational theory of the variety of GMV-algebras is decidable.

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MSC:

06D35 MV-algebras
06F15 Ordered groups
03B25 Decidability of theories and sets of sentences

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