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Perturbation analysis for denumerable Markov chains with application to queueing models.

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Summary: We study the parametric perturbation of Markov chains with denumerable state spaces. We consider both regular and singular perturbations. By the latter we mean that transition probabilities of a Markov chain, with several ergodic classes, are perturbed such that (rare) transitions among the different ergodic classes of the unperturbed chain are allowed. Singularly perturbed Markov chains have been studied in the literature under more restrictive assumptions such as strong recurrence ergodicity or Doeblin conditions. We relax these conditions so that our results can be applied to queueing models (where the conditions mentioned above typically fail to hold). Assuming ν-geometric ergodicity, we are able to explicitly express the steady-state distribution of the perturbed Markov chain as a Taylor series in the perturbation parameter. We apply our results to quasi-birth-and-death processes and queueing models.

MSC:

60J10 Markov chains (discrete-time Markov processes on discrete state spaces)
60J22 Computational methods in Markov chains
60J27 Continuous-time Markov processes on discrete state spaces
60K25 Queueing theory (aspects of probability theory)

Keywords:

geometric ergodicity; quasi-birth-and-death process; queueing model

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