Terwilliger, Paul; Vidunas, Raimundas

Leonard pairs and the Askey-Wilson relations. (English) Zbl 1062.33018

J. Algebra Appl. 3, No. 4, 411-426 (2004).

Let $\mathbb{K}$ denote a field and let V denote a vector space over $\mathbb{K}$ with finite positive dimension. An ordered pair of linear transformations $A: V \to V$ and $A^*: V \to V$ which satisfy the following two properties: (i) There exists a basis for V with respect to which the matrix representing A is irreducible tri-diagonal and the matrix representing A^* is diagonal, (ii) There exists a basis for V with respect to which the matrix representing A^* is irreducible tri-diagonal and the matrix representing A is diagonal, is called a Leonard pair on V. The authors show that there exists a sequence of scalars β, γ, γ^*, ρ, ρ^*, ω, η, η^* taken from $\mathbb{K}$ such that both

$$A^2A^* - \beta AA^*A + A^*A^2 - \gamma\,(AA^* + A^*A) - \rho\,A^* = \gamma^*A^2 + \omega A + \eta\,I,$$

and

$$A^{*2}A - \beta A^*AA^* + AA^{*2} - \gamma^*\,(A^*A + AA^*) - \rho^*A = \gamma A^{*2} + \omega A^* + \eta^*\,I.$$

The sequence is uniquely determined by the Leonard pair provided the dimension of V is at least 4. The equations above are called the Askey-Wilson relations.

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MSC:

33D45 Basic orthogonal polynomials and functions (Askey-Wilson polynomials, etc.)
05A30 q-calculus and related topics
15A03 Vector spaces, linear dependence, rank, lineability

Keywords:

Askey scheme; Askey-Wilson polynomial; q-Racah polynomial; Leonard pair; tridiagonal pair

Full Text: DOI arXiv

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