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Leonard pairs and the Askey-Wilson relations. (English) Zbl 1062.33018
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Let \mathbb{K} denote a field and let V denote a vector space over \mathbb{K} with finite positive dimension. An ordered pair of linear transformations $A : V \rightarrow V$ and $A^* : V \rightarrow V$ which satisfy the following two properties: (i) There exists a basis for V with respect to which the matrix representing A is irreducible tri-diagonal and the matrix representing A^* is diagonal, (ii) There exists a basis for V with respect to which the matrix representing A^* is irreducible tri-diagonal and the matrix representing A is diagonal, is called a Leonard pair on V . The authors show that there exists a sequence of scalars $\beta, \gamma, \gamma^*, \rho, \rho^*, \omega, \eta, \eta^*$ taken from \mathbb{K} such that both

$$A^2 A^* - \beta A A^* A + A^* A^2 - \gamma (A A^* + A^* A) - \rho A^* = \gamma^* A^2 + \omega A + \eta I,$$

and

$$A^{*2} A - \beta A^* A A^* + A A^{*2} - \gamma^* (A^* A + A A^*) - \rho^* A = \gamma A^{*2} + \omega A^* + \eta^* I.$$

The sequence is uniquely determined by the Leonard pair provided the dimension of V is at least 4. The equations above are called the Askey-Wilson relations.

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MSC:

- 33D45** Basic orthogonal polynomials and functions (Askey-Wilson polynomials, etc.)
- 05A30** q -calculus and related topics
- 15A03** Vector spaces, linear dependence, rank, lineability

Cited in **1** Review
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Keywords:

[Askey scheme](#); [Askey-Wilson polynomial](#); [\$q\$ -Racah polynomial](#); [Leonard pair](#); [tridiagonal pair](#)

Full Text: [DOI](#) [arXiv](#)

References:

- [1] DOI: 10.1137/0510092 · Zbl 0437.33014 · doi:10.1137/0510092
- [2] Bannai E., Algebraic Combinatorics I: Association Schemes (1984) · Zbl 0555.05019
- [3] DOI: 10.1016/S0012-365X(98)00196-4 · Zbl 0924.05067 · doi:10.1016/S0012-365X(98)00196-4
- [4] DOI: 10.1023/A:1008707417118 · Zbl 0967.05067 · doi:10.1023/A:1008707417118
- [5] Gasper G., Encyclopedia of Mathematics and its Applications 35, in: Basic hypergeometric series (1990)
- [6] Granovskii Ya. I., Zh. Èksper. Teoret. Fiz. 94 pp 49–
- [7] DOI: 10.1016/0003-4916(92)90336-K · Zbl 0875.17002 · doi:10.1016/0003-4916(92)90336-K
- [8] DOI: 10.1088/0305-4470/26/7/001 · Zbl 0784.17019 · doi:10.1088/0305-4470/26/7/001
- [9] F. A. Granbaum, The bispectral problem (Montreal, PQ, 1997) (Amer. Math. Soc., Providence, RI, 1998) pp. 31–45.
- [10] Granbaum F. A., Internat. Math. Res. Notices 8 pp 359–
- [11] DOI: 10.1016/0377-0427(95)00262-6 · Zbl 0865.33012 · doi:10.1016/0377-0427(95)00262-6
- [12] Grünbaum F. A., Comm. Math. Phys. 184 pp 173–
- [13] F. A. Grünbaum and L. Haine, Algebraic methods and q -special functions (Montréal, QC, 1996) (Amer. Math. Soc., Providence, RI, 1999) pp. 171–181.
- [14] DOI: 10.1017/CBO9780511569432.029 · doi:10.1017/CBO9780511569432.029
- [15] DOI: 10.1016/S0377-0427(99)00069-2 · Zbl 0926.33007 · doi:10.1016/S0377-0427(99)00069-2
- [16] T. Ito, K. Tanabe and P. Terwilliger, Codes and association schemes (Piscataway NJ, 1999), DIMACS Ser. Discrete Math. Theoret. Comput. Sci. 56 (Amer. Math. Soc., Providence RI, 2001) pp. 167–192. · Zbl 0995.05148

- [17] DOI: [10.1016/j.jpaa.2003.10.002](https://doi.org/10.1016/j.jpaa.2003.10.002) · Zbl [1037.17013](https://zbmath.org/?q=ser/1037.17013) · doi:[10.1016/j.jpaa.2003.10.002](https://doi.org/10.1016/j.jpaa.2003.10.002)
- [18] Koelink H. T., Acta Appl. Math. 44 pp 295–
- [19] DOI: [10.1090/S0002-9947-00-02588-5](https://doi.org/10.1090/S0002-9947-00-02588-5) · Zbl [0957.33014](https://zbmath.org/?q=ser/0957.33014) · doi:[10.1090/S0002-9947-00-02588-5](https://doi.org/10.1090/S0002-9947-00-02588-5)
- [20] DOI: [10.1137/S003614109630673X](https://doi.org/10.1137/S003614109630673X) · Zbl [0977.33013](https://zbmath.org/?q=ser/0977.33013) · doi:[10.1137/S003614109630673X](https://doi.org/10.1137/S003614109630673X)
- [21] DOI: [10.1007/s003659900118](https://doi.org/10.1007/s003659900118) · Zbl [0941.33011](https://zbmath.org/?q=ser/0941.33011) · doi:[10.1007/s003659900118](https://doi.org/10.1007/s003659900118)
- [22] DOI: [10.1137/0524049](https://doi.org/10.1137/0524049) · Zbl [0799.33015](https://zbmath.org/?q=ser/0799.33015) · doi:[10.1137/0524049](https://doi.org/10.1137/0524049)
- [23] DOI: [10.1137/0513044](https://doi.org/10.1137/0513044) · Zbl [0495.33006](https://zbmath.org/?q=ser/0495.33006) · doi:[10.1137/0513044](https://doi.org/10.1137/0513044)
- [24] Rosengren H., Centre for Mathematical Sciences, in: Multivariable orthogonal polynomials as coupling coefficients for Lie and quantum algebra representations (1999) · Zbl [0946.33013](https://zbmath.org/?q=ser/0946.33013)
- [25] DOI: [10.1023/A:1022494701663](https://doi.org/10.1023/A:1022494701663) · Zbl [0785.05089](https://zbmath.org/?q=ser/0785.05089) · doi:[10.1023/A:1022494701663](https://doi.org/10.1023/A:1022494701663)
- [26] DOI: [10.1023/A:1022480715311](https://doi.org/10.1023/A:1022480715311) · Zbl [0785.05090](https://zbmath.org/?q=ser/0785.05090) · doi:[10.1023/A:1022480715311](https://doi.org/10.1023/A:1022480715311)
- [27] DOI: [10.1023/A:1022415825656](https://doi.org/10.1023/A:1022415825656) · Zbl [0785.05091](https://zbmath.org/?q=ser/0785.05091) · doi:[10.1023/A:1022415825656](https://doi.org/10.1023/A:1022415825656)
- [28] DOI: [10.1016/S0024-3795\(01\)00242-7](https://doi.org/10.1016/S0024-3795(01)00242-7) · Zbl [0980.05054](https://zbmath.org/?q=ser/0980.05054) · doi:[10.1016/S0024-3795\(01\)00242-7](https://doi.org/10.1016/S0024-3795(01)00242-7)
- [29] DOI: [10.1142/9789812810199_0013](https://doi.org/10.1142/9789812810199_0013) · Zbl [1061.16033](https://zbmath.org/?q=ser/1061.16033) · doi:[10.1142/9789812810199_0013](https://doi.org/10.1142/9789812810199_0013)
- [30] DOI: [10.1216/rmjm/1030539699](https://doi.org/10.1216/rmjm/1030539699) · Zbl [1040.05030](https://zbmath.org/?q=ser/1040.05030) · doi:[10.1216/rmjm/1030539699](https://doi.org/10.1216/rmjm/1030539699)
- [31] Terwilliger P., J. Algebra
- [32] DOI: [10.1016/S0377-0427\(02\)00600-3](https://doi.org/10.1016/S0377-0427(02)00600-3) · Zbl [1035.05103](https://zbmath.org/?q=ser/1035.05103) · doi:[10.1016/S0377-0427\(02\)00600-3](https://doi.org/10.1016/S0377-0427(02)00600-3)
- [33] P. Terwilliger, Sūrikaiseikikenky ūsho Kōkyūroku 1109 (Algebraic combinatorics, Kyoto, 1999) pp. 67–79.
- [34] Terwilliger P., J. Comput. Appl. Math.
- [35] Terwilliger P., Geometric and Algebraic Combinatorics 2, Oisterwijk, The Netherlands (2002)
- [36] DOI: [10.1016/j.laa.2004.02.014](https://doi.org/10.1016/j.laa.2004.02.014) · Zbl [1075.05090](https://zbmath.org/?q=ser/1075.05090) · doi:[10.1016/j.laa.2004.02.014](https://doi.org/10.1016/j.laa.2004.02.014)
- [37] Zhedanov A. S., Teoret. Mat. Fiz. 89 pp 190–
- [38] DOI: [10.1142/S0217732392001269](https://doi.org/10.1142/S0217732392001269) · Zbl [1020.17514](https://zbmath.org/?q=ser/1020.17514) · doi:[10.1142/S0217732392001269](https://doi.org/10.1142/S0217732392001269)

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