

**Khrennikov, Andrei Yu.; Nilson, Marcus**

***p*-adic deterministic and random dynamics.** (English) Zbl 1135.37003

Mathematics and its Applications (Dordrecht) 574. Dordrecht: Kluwer Academic Publishers (ISBN 1-4020-2659-5/hbk). xviii, 270 p. (2004).

The book is devoted to non-archimedean analogs of problems of holomorphic dynamics. Publications on this subject can be divided into two categories – works on qualitative behavior of general dynamical systems, usually based on deep results of arithmetic algebraic geometry, and investigations of some specific dynamical systems where relatively elementary number-theoretic techniques bring a more or less complete description of all essential features. The authors of this book are among the most active contributors to activities of the latter type, and their results constitute the main material of the book.

The book begins with a brief historical survey (Chapter 1) and the introductory material from *p*-adic analysis (Chapter 2). In Chapter 3 the authors study in detail monomial dynamical systems in field  $\mathbb{Q}_p$ , its finite extensions, and in  $\mathbb{C}_p$  (periodic points, cycles, attractors and repellers, Siegel disks etc). Chapter 4 is devoted to perturbations of monomial dynamical systems, while Chapter 5 deals with some examples and general results regarding polynomial dynamical systems in finite extensions of  $\mathbb{Q}_p$ . An introduction to the dynamics of power series is given in Chapter 6. Chapter 7 is devoted to ergodic behavior of *p*-adic monomial dynamical systems.

The next two chapters (8 and 9) present various attempts by Khrennikov and his collaborators to develop *p*-adic models of mental processes – dynamical systems appear in the theory of *p*-adic neural networks, the *p*-adic dynamical model of memory and a model of associative thinking. These conjectural applications were among the motivations to develop *p*-adic random dynamics (Chapter 10). The latter subject is related to diffusion on trees, and a sketch of its possible applications given in Chapter 11 leads a reader to pseudo-differential operators and wavelets on the boundary of a tree (Chapter 12).

Finally, in Chapter 13 the authors give an introduction to the theory of *p*-adic-valued probabilities. For further material on this subject see earlier books by *A. Yu. Khrennikov*, in particular “Non-archimedean analysis: quantum paradoxes, dynamical systems and biological models”. Mathematics and its Applications, 427. Dordrecht: Kluwer Academic Publishers (1997; [Zbl 0920.11087](#)); “Interpretations of probability”. Utrecht: VSP (1999; [Zbl 0998.81508](#)).

The book will be of interest both for specialists in dynamical systems wishing to see *p*-adic aspects of their field, and readers looking for new applications of mathematics, even if these applications are still in their hypothetical stage.

Reviewer: [Anatoly N. Kochubei \(Kyïv\)](#)

**MSC:**

- 37-02** Research exposition (monographs, survey articles) pertaining to dynamical systems and ergodic theory
- 37F99** Dynamical systems over complex numbers
- 11S85** Other nonanalytic theory
- 28C10** Set functions and measures on topological groups or semigroups, Haar measures, invariant measures
- 28D15** General groups of measure-preserving transformations
- 37H99** Random dynamical systems
- 37N25** Dynamical systems in biology
- 60A05** Axioms; other general questions in probability
- 92B20** Neural networks for/in biological studies, artificial life and related topics

Cited in **2** Reviews  
Cited in **36** Documents

**Keywords:**

*p*-adic numbers; *p*-adic dynamical system; conjugate maps; *p*-adic neural network; Markovian dynamics; *p*-adic mental spaces; ultrametric wavelet; *p*-adic probability