Karlsen, K. H.; **Towers, J. D.**

Convergence of the Lax-Friedrichs scheme and stability for conservation laws with a discontinuous space-time dependent flux. (English) Zbl 1112.65085

Chin. Ann. Math., Ser. B 25, No. 3, 287-318 (2004).

The paper presents a convergence proof for the Lax-Friedrichs finite difference scheme in the context of non-convex genuinely nonlinear scalar conservation laws of the form

$$u_t + f(k(x,t),u)_x = 0,$$

where the coefficient $k(x,t)$ is allowed to be discontinuous along curves in the (x,t) plane. It is shown that a convergent subsequence of approximations produced by the Lax-Friedrichs scheme converges to an entropy solution, implying that the entire computed sequence converges.

Reviewer: Andreas Meister (Kassel)

MSC:

65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

35L65 Hyperbolic conservation laws

Keywords:

conservation laws; Lax-Friedrichs scheme; entropy condition; compensated compactness; nonconvex fluxes

Full Text: DOI

References:

[1] G. Q. Chen, Some Current Topics on Nonlinear Conservation Laws (Amer. Math. Soc., Providence, RI, 2000) pp. 33–75.

[2] P. D. Lax, Contributions to Nonlinear Functional Analysis, ed. E. Zarantonello (Academic Press, New York, 1971) pp. 603–634.

[3] L. Tartar, Nonlinear Analysis and Mechanics: Heriot-Watt Symposium IV (Pitman, Boston, Mass., 1979) pp. 136–212.

[4] L. Tartar, Systems of Nonlinear Partial Differential Equations (Oxford, 1982), NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci. 111 (Reidel, Dordrecht, 1983) pp. 263–285.