Runde, Volker
Dual Banach algebras: Connes-amenability, normal, virtual diagonals, and injectivity of the predual bimodule. (English) ⟨Zbl 1087.46035⟩
Math. Scand. 95, No. 1, 124-144 (2004).

Let A be a Banach algebra. A is called a dual Banach algebra if it is a dual Banach A-module. Dual Banach algebras include every von Neumann algebra and the measure algebra $M(G)$ of a locally compact group G. A dual Banach algebra A is called Connes-amenable if every weak*-continuous derivation from A into a normal, dual Banach A-bimodule is inner. Connes-amenability was introduced by *B. E. Johnson, R. V. Kadison* and *J. Ringrose* for von Neumann algebras [Bull. Soc. Math. Fr. 100, 73–96 (1972; Zbl 0234.46066)]. Connes-amenability is equivalent to injectivity and semidiscreteness for von Neumann algebras.

For the measure algebra $M(G)$, it was shown by the author that the measure algebra $M(G)$ is Connes-amenable if and only if G is compact [J. Lond. Math. Soc., II. Ser. 67, No. 3, 643–656 (2003; Zbl 1040.22002)]. In the present paper, the author considers the following three properties of a dual Banach algebra: (i) A is Connes-amenable; (ii) A has a normal, virtual diagonal; (iii) A_* is an injective A-bimodule. He shows that (iii) implies (ii) and thus (i). However, the converse need not hold in general. This is done by considering the measure algebra $M(G)$ of an infinite amenable locally compact group G. It answers a question of A. Ya. Helemskii. These conditions are also studied for the Fourier Stieltjes algebra $B(G)$ for certain G.

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MSC:

46H05	General theory of topological algebras
43A10	Measure algebras on groups, semigroups, etc.
43A20	L^1-algebras on groups, semigroups, etc.

Cited in **3** Reviews
Cited in **18** Documents

Keywords:

dual Banach algebras; amenability; virtual diagonal; injectivity; measure algebra; Fourier Stieltjes algebra

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