Grüne, Lars

Error estimation and adaptive discretization for the discrete stochastic Hamilton-Jacobi-Bellman equation. (English) Zbl 1074.65009

Numer. Math. 99, No. 1, 85-112 (2004).

The dynamic programming method is a well known technique for the numerical solution of optimal control problems. Generalizing the technique and results from the deterministic case [cf. the author, ibid. 75, 319–337 (1997; Zbl 0880.65045)], the author obtains a posteriori error estimates for the space discretization of the stochastic Hamilton-Jacobi-Bellman equation. This method gives full global information about the optimal value function of the related stochastic optimal control problem. Therefore a feedback optimal control can be obtained.

It is also demonstrated that the a posteriori error estimates are efficient and reliable for the numerical approximation of PDEs and they allow to derive a bound for the numerical error corresponding to the derivatives. The asymptotic behavior of the error estimates with respect to the size of the grid elements is also investigated. Finally, an adaptive space discretization scheme is developed and numerical examples are presented.

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MSC:

65C30	Numerical solutions to stochastic differential and integral equations
60H15	Stochastic partial differential equations (aspects of stochastic analysis)
60H35	Computational methods for stochastic equations (aspects of stochastic analysis)
49J55	Existence of optimal solutions to problems involving randomness
65K10	Numerical optimization and variational techniques
49M25	Discrete approximations in optimal control
65N15	Error bounds for boundary value problems involving PDEs
49L20	Dynamic programming in optimal control and differential games

Keywords:

stochastic optimal control; stochastic Hamilton-Jacobi-Bellman equation; a posteriori error estimates; feedback optimal control; numerical examples

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