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**Schrödinger and Fock representation for a field theory on curved spacetime.** (English)

Zbl 1074.81054

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This paper deals with linear free field theories in curved (globally hyperbolic) spacetime. In particular, the work focus on the interplay of two different approaches used in the literature: Fock space methods and the Schroedinger functional description. The precise sense in which the two representations are related is carefully discussed. Several properties of these representations are studied, among them the well known fact that the Schrödinger counterpart of the usual Fock representation is described by a Gaussian measure. A real scalar field theory is considered, both on Minkowski spacetime for arbitrary, non-inertial embeddings of the Cauchy surface, and for arbitrary (globally hyperbolic) curved spacetimes. As a concrete example, the Schroedinger representation on stationary and homogeneous cosmological spacetimes is constructed. As a final remark I would like to notice that, despite the paper presents some interesting observations, there is an important point which is not discussed as it would deserve in the work. The construction of Fock structure is presented in terms of a suitable complex structure in a (symplectic) space of solutions of the classical field equations. As is well-known, such an approach works very well in Minkowski spacetime but it turns out to be quite involved in a general spacetime [see discussion in section 3.2 of *R. M. Wald, Quantum Field Theory in Curved Space-Time and Black Hole Thermodynamics*, (Chicago University Press, Chicago) (1994; Zbl 0842.53052)]. Moreover treatment of nonpure states is not very trivial in this way, since complex structures used as presented in the paper select pure states only barring nontrivial re-adaptations [see the proof of Proposition 3.1 in *B. S. Kay and R. M. Wald, Phys. Rep.* 207, No. 2, 49–136 (1991; Zbl 0861.53074)].

Reviewer: [Valter Moretti \(Povo\)](#)

#### MSC:

- [81T20](#) Quantum field theory on curved space or space-time backgrounds
- [81T10](#) Model quantum field theories
- [81R15](#) Operator algebra methods applied to problems in quantum theory
- [51M10](#) Hyperbolic and elliptic geometries (general) and generalizations
- [58J45](#) Hyperbolic equations on manifolds
- [81S05](#) Commutation relations and statistics as related to quantum mechanics (general)

Cited in **6** Documents

#### Keywords:

[linear field theory in curved spacetime](#); [Fock representation](#); [Schrödinger representation](#)

**Full Text:** [DOI](#) [arXiv](#)

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