

**Ait-Haddou, R.; Jinha, A.; Herzog, W.; Binding, P.**

**Analysis of the force-sharing problem using an optimization model.** (English) Zbl 1053.92005  
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Summary: In biomechanics, one frequently used approach for finding a unique set of muscle forces in the ‘force-sharing problem’ is to formulate and solve a nonlinear optimization problem of the form:  $\min \varphi(f) = \sum (f_i/\omega_i)^\alpha$  subject to  $Af = b$  and  $f \geq 0$ . Solutions to this problem have typically been obtained numerically for complex models, or analytically for specific musculoskeletal geometries. Here, we present simple geometrical methods for analyzing the solution to this family of optimization problems for a general  $n$ -degrees-of-freedom musculoskeletal system.

For example, it is shown that the moment-arm vectors of active ( $f_i > 0$ ) and passive ( $f_i = 0$ ) muscles are separated by a hyperplane through the origin of the moment-arm vector space. For the special case of a system with two degrees-of-freedom, solutions can be readily represented in graphical form. This allows for powerful interpretations of force-sharing calculated using optimization.

**MSC:**

[92C10](#) Biomechanics

[49N90](#) Applications of optimal control and differential games

Cited in **3** Documents

**Keywords:**

[Muscle](#); [Force-sharing](#); [Optimization](#); [Movement control](#); [Agonist](#); [Antagonist](#)

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