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Global in time weak solutions for compressible barotropic self-gravitating fluids. (English)

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The Navier-Stokes-Poisson system:

$$\rho_t + \operatorname{div}(\rho \vec{u}) = 0, \tag{1}$$

$$(\rho u^i)_t + \operatorname{div}(\rho u^i \vec{u}) + p_{x_i} = \mu \Delta u^i + (\lambda + \mu)(\operatorname{div} \vec{u})_{x_i} + G\rho \partial_x \Phi,\ i = 1, 2, 3, \tag{2}$$

$$-\Delta \Phi = \rho + g \tag{3}$$

describing the time evolution of the density $\rho = \rho(t, x)$ and the velocity $\vec{u} = \vec{u}(t, x)$ of a gaseous star is considered.

Here p is the pressure, Φ is the gravitational potential of the star, G is a positive constant, $g = g(x)$ is a given function, and the viscosity coefficients μ and λ satisfy conditions: $\mu > 0$, $\lambda + \frac{2}{3}\mu \geq 0$.

The system (1)–(3) is considered with the initial conditions:

$$\rho(0) = \rho_0, \quad (\rho u^i)(0) = q^i, \quad i = 1, 2, 3 \tag{4}$$

and the no-slip boundary conditions for the velocity:

$$u^i|_{\partial\Omega} = 0, \quad i = 1, 2, 3. \tag{5}$$

The main result of this paper states that if the data ρ_0, q^i, $i = 1, 2, 3$ satisfy the compatibility conditions, the pressure p is given by the barotropic constitutive law and g belongs to the class $L^1 \cap L^\infty(\mathbb{R}^3)$, then there exists a global in time weak solution ρ, $\vec{u}$ of the problem (1)–(5).

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MSC:

35Q30	Navier-Stokes equations
85A30	Hydrodynamic and hydromagnetic problems in astronomy and astrophysics
76N10	Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics
35D05	Existence of generalized solutions of PDE (MSC2000)

Keywords:
compressible self-gravitating fluid; Navier-Stokes-Poisson system; global in time solutions; weak solution

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