

**Wihler, Thomas P.; Frauenfelder, Philipp; Schwab, Christoph**

**Exponential convergence of the  $hp$ -DGFEM for diffusion problems.** (English) Zbl 1059.65095  
*Comput. Math. Appl.* 46, No. 1, 183-205 (2003).

Two different formulations of the  $hp$ -discontinuous Galerkin finite element method (DGFEM) are considered for the two-dimensional stationary diffusion problem. As in the usual FEM, mesh refinement strategy is important when corner singularity caused by polygonal shape of domains exists. The authors prove exponential convergence of the  $hp$ -version of DGFEM on geometrically refined meshes in polygons. Several variants of interior penalization are covered. Numerical experiments indicate the sharpness of the theoretical results as well as the weak dependence of the DGFEM approximation on the particular choice of interior penalization and the penalty parameter. In certain cases, stabilization techniques are effective.

Reviewer: [Fumio Kikuchi \(Tokyo\)](#)

**MSC:**

- [65N12](#) Stability and convergence of numerical methods for boundary value problems involving PDEs
- [65N30](#) Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
- [65N50](#) Mesh generation, refinement, and adaptive methods for boundary value problems involving PDEs
- [35J25](#) Boundary value problems for second-order elliptic equations

Cited in **1** Review  
Cited in **17** Documents

**Keywords:**

[FEM](#); [discontinuous Galerkin methods](#); [exponential convergence](#); [diffusion problems](#); [corner singularities](#); [finite element method](#); [mesh refinement](#); [interior penalization](#); [numerical experiments](#); [stabilization](#)

**Full Text:** [DOI](#)

**References:**

- [1] Reed, W.H.; Hill, T.R., Triangular mesh methods for the neutron transport equation, ()
- [2] Lesaint, P.; Raviart, P.A., On a finite element method for solving the neutron transport equation, (), 89-123
- [3] Cockburn, B.; Shu, C.-W., The local discontinuous Galerkin method for time-dependent reaction-diffusion systems, *SIAM J. numer. anal.*, 35, 2440-2463, (1998) · [Zbl 0927.65118](#)
- [4] Cockburn, B.; Shu, C.-W., The Runge-Kutta discontinuous Galerkin method for conservation laws: multidimensional systems, *J. comput. phys.*, 141, 199-244, (1998) · [Zbl 0920.65059](#)
- [5] (), 451-458
- [6] Johnson, C.; Nävert, U.; Pitkäranta, J., Finite element methods for linear hyperbolic problems, *Comp. meth. appl. mech. enrg.*, 45, 285-312, (1984) · [Zbl 0526.76087](#)
- [7] Johnson, C.; Pitkäranta, J., An analysis of the discontinuous Galerkin method for a scalar hyperbolic equation, *Math. comp.*, 46, 1-26, (1986) · [Zbl 0618.65105](#)
- [8] Falk, R.S.; Richter, G.R., Local error estimates for a finite element method for hyperbolic and convectiondiffusion equations, *SIAM J. numer. anal.*, 29, 730-754, (1992) · [Zbl 0757.65104](#)
- [9] Peterson, T.E., A note on the convergence of the discontinuous Galerkin method, *Math. comp.*, 28, 133-140, (1991) · [Zbl 0729.65085](#)
- [10] Richter, G., An optimal-order error estimate for the discontinuous Galerkin method, *Math. comp.*, 50, 75-88, (1988) · [Zbl 0643.65059](#)
- [11] Flaherty, J.E.; Loy, R.M.; Shephard, M.S.; Teresco, J.D., Software for parallel adaptive solution of conservation laws by discontinuous Galerkin methods, (), 113-123 · [Zbl 0946.65089](#)
- [12] Arnold, D.N.; Brezzi, F.; Cockburn, B.; Marini, D., An interior penalty finite element method with discontinuous elements, (), 89-101
- [13] Wheeler, M.F., An elliptic collocation finite element method with interior penalties, *SIAM J. numer. anal.*, 15, 152-161, (1978) · [Zbl 0384.65058](#)

- [14] Rivière, B.; Wheeler, M.F.; Girault, V., Improved energy estimates for interior penalty, constrained and discontinuous Galerkin methods for elliptic problems. part I, Computational geosciences, 3/4, 337-360, (1999) · [Zbl 0951.65108](#)
- [15] Oden, J.T.; Babuška, I.; Baumann, C., A discontinuous hp-FEM for diffusion problems, J. comput. phys., 146, 491-519, (1998) · [Zbl 0926.65109](#)
- [16] Richter, G., The discontinuous Galerkin method with diffusion, Math. comp., 58, 631-643, (1992) · [Zbl 0783.65078](#)
- [17] Houston, P.; Schwab, C.; Süli, E., Discontinuous hp-finite element methods for advection-diffusion problems, (), 2000-2007
- [18] P. Houston, C. Schwab and E. Süli, Stabilized  $\text{hp}$ -finite element methods for first-order hyperbolic problems SIAM J. Numer. Anal. (to appear).
- [19] Babuška, I.; Guo, B.Q., Regularity of the solutions of elliptic problems with piecewise analytic data, part I, SIAM J. math. anal., 19, 172-203, (1988) · [Zbl 0647.35021](#)
- [20] Babuška, I.; Guo, B.Q., Regularity of the solutions of elliptic problems with piecewise analytic data, part II, SIAM J. math. anal., 20, 763-781, (1989) · [Zbl 0706.35028](#)
- [21] Schwab, C., P- and hp-finite element methods. theory and applications to solid and fluid mechanics, (1998), Oxford University Press · [Zbl 0910.73003](#)
- [22] Baumann, C., An  $\text{hp}$ -adaptive discontinuous Galerkin FEM for computational fluid dynamics, ()
- [23] Bey, K.S.; Oden, J.T.,  $\text{hp}$ -version discontinuous Galerkin methods for hyperbolic conservation laws, Comput. methods appl. mech. engrg., 133, 259-286, (1996) · [Zbl 0894.76036](#)
- [24] P. Castillo, B. Cockburn, I. Perugia and D. Schötzau, An a  $\text{priori}$  error analysis of the local discontinuous Galerkin method for elliptic problems, SIAM J. Numer. Anal., (submitted).
- [25] B. Cockburn, G. Kanschat, I. Perugia and D. Schötzau, Superconvergence of the local discontinuous Galerkin method for elliptic problems on Cartesian grids,  $\text{SIAM J. Numer. Anal.}$ , (submitted). · [Zbl 1041.65080](#)
- [26] Guo, B.Q.; Babuška, I., On the regularity of elasticity problems with piecewise analytic data, Adv. appl. math., 14, 307-347, (1993) · [Zbl 0790.35028](#)
- [27] Karniadakis, G.E.; Sherwin, S., Spectral/hp finite element methods in CFD, (1999), Oxford University Press
- [28] Melenk, J.M.; Schwab, C., An hp finite element method for convection-diffusion problems, () · [Zbl 0952.65061](#)
- [29] Nitsche, J., Über ein variationsprinzip zur Lösung von Dirichlet problemen bei verwendung von teilräumen, die keinen randbedingungen unterworfen sind, (), 9-15 · [Zbl 0229.65079](#)
- [30] Stroustrup, B., The C++ programming language, (1991), Addison-Wesley Massachusetts
- [31] Szabó, B.A.; Babuška, I., Finite element analysis, (1991), Wiley New York
- [32] Wihler, T.P., Discontinuous Galerkin FEM for elliptic problems in polygonal domains, ETH Zürich, diss. no. 14973, (2003), Also available online:
- [33] Wihler, T.P.; Schwab, C., Robust exponential convergence of the hp discontinuous Galerkin FEM for convection-diffusion problems in one space dimension, East-west J. numer. math., 8, 1, 57-70, (2000) · [Zbl 0958.65090](#)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.