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**Local asymptotics for polynomial spline regression.** (English) Zbl 1042.62035  
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Summary: We develop a general theory of local asymptotics for least-squares estimates over polynomial spline spaces in a regression problem. The polynomial spline spaces we consider include univariate splines, tensor product splines, and bivariate or multivariate splines on triangulations. We establish asymptotic normality of the estimate and study the magnitude of the bias due to spline approximation. The asymptotic normality holds uniformly over the points where the regression function is to be estimated and uniformly over a broad class of design densities, error distributions and regression functions.

The bias is controlled by the minimum  $L_\infty$  norm of the error when the target regression function is approximated by a function in the polynomial spline space that is used to define the estimate. The control of bias relies on the stability in  $L_\infty$  norm of  $L_2$  projections onto polynomial spline spaces. Asymptotic normality of least-squares estimates over polynomial or trigonometric polynomial spaces is also treated by the general theory. In addition, a preliminary analysis of additive models is provided.

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**MSC:**

**62G08** Nonparametric regression and quantile regression  
**62G20** Asymptotic properties of nonparametric inference

Cited in **70** Documents

**Keywords:**

[asymptotic normality](#); [least squares](#); [polynomial regression](#); [regression splines](#)

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