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On the growth of solutions of a class of higher order differential equations. (English)

Zbl 1056.30029
Acta Math. Sci., Ser. B, Engl. Ed. 24, No. 1, 52-60 (2004).

Let $f(z)$ be a meromorphic function in the complex plane. Denote by $\sigma(f)$ the growth order of $f(z)$ and define a hyper-order of $f(z)$ by

$$\sigma_2(f) = \limsup_{r\to\infty} \log\log T(r,f)/\log r,$$

where $T(r,f)$ is the characteristic function of $f(z)$. Let $H_j(z)$, $j=0,1,\dots,k-1$ be entire functions. The authors study linear differential equations of the form

$$f^{(k)} + H_{k-1}f^{(k-1)} + \cdots + H_s f^{(s)} + \cdots + H_0 f = 0.$$

The "one dominate coefficient" case below is treated in this article. Let $h_j(z)$, $j=0,1,\dots,k-1$ be entire functions with $\sigma(h_j)<1$, and $H_j(z) = h_j(z)e^{a_j z}$, $j=0,1,\dots,k-1$, where a_j, $j=0,1,\dots,k-1$ are complex numbers. They suppose that there exist a_s such that $h_s(z)\not\equiv 0$, and for $j\neq s$ if $H_j(z)\not\equiv 0$, $a_j = c_j a_s$, $0<c_j<1$; if $H_j(z)\equiv 0$, define $c_j=0$. Statements of their results are the following. Every transcendental solution of the differential equation above satisfies $\sigma(f)=\infty$. Further, if $h_j(z)$ are polynomials, then $\sigma(f)=\infty$ and $\sigma_2(f)=1$. Main tools for the proofs are the Nevanlinna theory and the Wiman-Valiron theory. In particular, estimates for logarithmic derivatives of meromorphic functions due to *G. G. Gundersen* [J. Lond. Math. Soc., II. Ser. 37, No. 1, 88–104 (1988; Zbl 0638.30030)] play important roles.

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MSC:

30D35 Value distribution of meromorphic functions of one complex variable, Nevanlinna theory

34M10 Oscillation, growth of solutions to ordinary differential equations in the complex domain

Keywords:
complex oscillation; growing solutions; hyper-order

Full Text: DOI