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A topological characterisation of relatively hyperbolic groups. (English) Zbl 1043.20020
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The notion of a relatively hyperbolic group was introduced by M. Gromov and has been elaborated on in various papers. As hyperbolic groups generalize fundamental groups of compact hyperbolic manifolds, the relatively hyperbolic groups generalize fundamental groups of hyperbolic manifolds of finite volume. The paper gives dynamical and topological properties characterising relatively hyperbolic groups in terms of their boundaries.

The following theorem is proved: Suppose that M is a non-empty, perfect and metrisable compactum, and Γ is a convergence group acting on M such that M consists only of conical limit points and bounded parabolic points. Suppose also that the quotient of the set of bounded parabolic points by Γ is finite and the stabiliser of each bounded parabolic point is finitely generated. Then Γ is hyperbolic relative to the set of its maximal parabolic subgroups and M is equivariantly homeomorphic to the boundary of Γ .

The main idea of the proof of the above theorem is the following. The author constructs a “system of annuli”, generalizing one given in a paper of *B. H. Bowditch* [*J. Am. Math. Soc.* 11, No. 3, 643-667 (1998; [Zbl 0906.20022](#))], which gives rise to a path hyperbolic quasimetric on the set of distinct triples union of the set of bounded parabolic points. Next, using the geometrically finite action the author constructs a graph satisfying all properties of relative hyperbolicity.

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MSC:

- [20F67](#) Hyperbolic groups and nonpositively curved groups
- [20F65](#) Geometric group theory
- [57M07](#) Topological methods in group theory
- [57S05](#) Topological properties of groups of homeomorphisms or diffeomorphisms

Cited in **2** Reviews
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[relatively hyperbolic groups](#); [convergence groups](#); [conical limit points](#); [bounded parabolic points](#)

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