Karlsen, K. H.; **Risebro, N. H.**; **Towers, J. D.**

L^1 stability for entropy solutions of nonlinear degenerate parabolic convection-diffusion equations with discontinuous coefficients. (English) Zbl 1036.35104

Skr., K. Nor. Vidensk. Selsk. 2003, No. 3, 49 p. (2003).

Entropy conditions of Kruzhkov-type and uniqueness for nonlinear degenerate parabolic convection-diffusion initial value problems of the type

$$u_t + f(\gamma(x), u)_x = A(u)_{xx}, \quad (x,t) \in \mathbb{R} \times (0,T),$$

$$u(x,0) = u_0(x), \quad x \in \mathbb{R},$$

are investigated in this paper. Here $u(x,t)$ is an unknown function, A is nondecreasing, so this problem becomes as mixed hyperbolic-parabolic one. Only degeneration in a finite collection of disjoint intervals for A is admitted in this article. The flux function f is supposed to be Lipschitz continuous in each variable, and γ is assumed to be piecewise C^1 with finitely many jumps.

The main goal of this paper is to prove that weak solutions satisfying the entropy condition (entropy solutions) are stable in the L^1 norm and thus unique. This can be done only if two aditional conditions are fulfilled. One of them is the so-called cross condition for the convective flux function, and the second is a technical assumption for the existence of the right and left traces of the solution in case of jumps in γ. In some important cases the existence of traces can be proved directly from the entropy condition. Using the Engquist-Osher finite-difference scheme and for the hyperbolic case the front tracking method, it is proved that the convergent limits of this scheme satisfy an analogous Kruzhkov type entropy inequality.

From the authors' previous existence results, combined with the L^1 stability proved in this paper, it is shown, that for some important cases the studied initial value problem is well posed.

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MSC:

35K65 Degenerate parabolic equations
35K57 Reaction-diffusion equations
35B35 Stability in context of PDEs
35R05 PDEs with low regular coefficients and/or low regular data

Keywords:

conservation law; cross condition; uniqueness; existence; difference scheme; front tracking