Bruin, Nils
Chabauty methods using elliptic curves. (English) Zbl 1135.11320
J. Reine Angew. Math. 562, 27-49 (2003).

From the text: Since 1983 [*G. Faltings*, Invent. Math. 73, No. 3, 349–366 (1983; Zbl 0588.14026)], it is known that an algebraic curve of genus $g \geq 2$ over a number field has only finitely many rational points. The proof of this theorem does not help in actually determining them, however. A much older, partial, proof by *C. Chabauty* [see C. R. Acad. Sci., Paris 212, 1022–1024 (1941; Zbl 0025.24903 and JFM 67.0105.02)] does suggest a way of bounding the number of rational points.

In this article, we consider algebraic curves over $\mathbb{Q}$ that cover an elliptic curve over some extension of $\mathbb{Q}$. We show how we can use the arithmetic on that elliptic curve to obtain information on the rational points on the cover. We apply this method to curves arising from the Diophantine equations $x^2 \pm y^4 = z^5$ and $x^8 + y^3 = z^2$ and determine all solutions with coprime, integral x, y, z. To do this, we determine the rational points on several curves of genus 5 and 17.

MSC:

11G05 Elliptic curves over global fields
14G05 Rational points

Keywords:

finitely many rational points; elliptic curve

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