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Connective coverings of spaces of holomorphic maps. (English) Zbl 1031.58005

Osaka J. Math. 40, No. 3, 741-750 (2003).

Let $\mathrm{Hol}_d(S^2,\mathbb{C}p^n)$ and $\mathrm{Hol}^*_d(S^2,\mathbb{C}P^n)$ be the spaces of holomorphic maps and based holomorphic maps $S^2\to\mathbb{C}P^n$ of degree d. Let $h_n: S^{2n+1}\to\mathbb{C}P^n$ be the Hopf fibering with fibre S^1 and

$$\widetilde{\mathrm{Hol}}_d(S^2,\mathbb{C}P^n)=\big\{(f,x)\in\mathrm{Hol}_d(S^2,\mathbb{C}P^n)\times S^{2n+1}: ev(f)=h_n(x)\big\}.$$

In [*K. Yamaguchi*, Kyushu J. Math. 56, 381-389 (2002; Zbl 1041.55005)] the author showed $\widetilde{\mathrm{Hol}}_d(S^2,S^2)$ is the universal covering of $\mathrm{Hol}_d(S^2,S^2)$ and homotopy equivalent to $\widetilde{\mathrm{Hol}^*_d}\times S^3$, where $\widetilde{\mathrm{Hol}^*_d}$ is the universal covering of $\mathrm{Hol}^*(S^2,S^2)$. In this paper, assuming $n\geq 2$ and $d\geq 1$, $\widetilde{\mathrm{Hol}}_d(S^2,\mathbb{C}P^n)$ is shown to be the 2-connected covering of $\mathrm{Hol}_d(S^2,\mathbb{C}P^n)$. Existence of a fibration sequence

$$\mathrm{Hol}^*_d(S^2,\mathbb{C}P^n)@>\widetilde{j}_d>>\widetilde{\mathrm{Hol}}_d(S^2,\mathbb{C}P^n)@>\widetilde{ev}>>S^{2n+1},$$

is also shown (Th. 1.3). This fibration have a section if and only if $n\equiv$ ‘ (mod 2) or $n\equiv d\equiv O\pmod 2$, by the results on Whitehead product of complect projective spaces [*G. W. Whitehead*, Ann. Math. (2) 47, 460-475 (1946; Zbl 0060.41106)]. But there are isomorphisms of graded Abelian groups and graded rings

$$H_*\big(\widetilde{\mathrm{Hol}}_d(S^2,\mathbb{C}P^n),A\big)\cong H_*\big(\mathrm{Hol}^*_d(S^2,\mathbb{C}P^n),A\big)\otimes H_*(S2n+1,A),$$

$$H^*\big(\widetilde{\mathrm{Hol}}_d(S^2,\mathbb{C}P^n),A\big)\cong H^*\big(\mathrm{Hol}^*_d(S^2,\mathbb{C}p^n),A\big)\otimes H^*(S^{2n+1},A),$$

where A is an Abelian group (Prop. 1.5), as a consequence of the computation of the homology of double loop space of S^{2n+} Lemma 3.1, cf. *F. R. Cohen*, *T. I. Lada* and *J. P. May*, [‘The homology of iterated loop spaces’, Lect. Notes Math. 533 (1976; Zbl 0334.55009)]. These are proved in Sect. 2 and 3. In Sect. 4, the last Section, homotopy types of $\mathrm{Hol}_1(S^2,\mathbb{C}P^n)$ and $\widetilde{\mathrm{Hol}}_1(S^2,\mathbb{C}P^n)$ are determined explicitely (Th. 1.6), analyzing U_{n+1}-action on $\mathrm{Hol}_1(\mathbb{C}p^k,\mathbb{C}P^n)$ induced from the U_{n+1}-action on $\mathbb{C}P^n$. For example, the followings are shown

$$\widetilde{\mathrm{Hol}}_1(S^2,\mathbb{C}P^2)\simeq SU_3,\quad \widetilde{\mathrm{Hol}}_1(S^2,\mathbb{C}P^2)\simeq S^5\times S^7.$$

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MSC:

58D15 Manifolds of mappings
55P35 Loop spaces
55R05 Fiber spaces in algebraic topology
32H99 Holomorphic mappings and correspondences

Keywords:

space of holomorphic maps; fibration sequence; Whitehead product; evaluation map; Hopf fibering