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Numerical methods for the generalized Zakharov system. (English) Zbl 1236.76043
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Summary: We present two numerical methods for the approximation of the generalized Zakharov system (ZS). The first one is the time-splitting spectral (TSSP) method, which is explicit, time reversible, and time transverse invariant if the generalized ZS is, keeps the same decay rate of the wave energy as that in the generalized ZS, gives exact results for the plane-wave solution, and is of spectral-order accuracy in space and second-order accuracy in time. The second one is to use a local spectral method, the discrete singular convolution (DSC) for spatial derivatives and the fourth-order Runge–Kutta (RK4) for time integration, which is of high (the same as spectral)-order accuracy in space and can be applied to deal with general boundary conditions. In order to test accuracy and stability, we compare these two methods with other existing methods: Fourier pseudospectral method (FPS) and wavelet-Galerkin method (WG) for spatial derivatives combining with the RK4 for time integration, as well as the standard finite difference method (FD) for solving the ZS with a solitary-wave solution. Furthermore, extensive numerical tests are presented for plane waves, solitary-wave collisions in 1d, as well as a 2d problem of the generalized ZS. Numerical results show that TSSP and DSC are spectral-order accuracy in space and much more accurate than FD, and for stability, TSSP requires $k = O(h)$, DSC–RK4 requires $k = O(h^2)$ for fixed acoustic speed, where k is the time step and h is the spatial mesh size.

MSC:

76M22 Spectral methods applied to problems in fluid mechanics
76X05 Ionized gas flow in electromagnetic fields; plasmic flow
65M70 Spectral, collocation and related methods for initial value and initial-boundary value problems involving PDEs

Full Text: DOI

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