Alon, Noga; **Ding, Guoli**; **Oporowski, Bogdan**; **Vertigan, Dirk**
Partitioning into graphs with only small components. (English) Zbl 1023.05045
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The authors prove several results on edge partitions and vertex partitions of graphs into graphs with bounded size components. The main results are:

(1) Every graph with maximum degree at most Δ and tree-width at most k admits a vertex partition into two induced subgraphs G_1, G_2 such that each connected component of G_1 and G_2 has at most $24k\Delta$ vertices, and an edge partition into two subgraphs H_1, H_2 such that each connected component of H_1 and H_2 has at most $24k\Delta(\Delta+1)$ vertices.

(2) Every graph with maximum degree $\Delta \geq 3$ admits a vertex partition into $\lfloor \frac{\Delta+2}{3} \rfloor$ induced subgraphs G_i such that each connected component of G_i has at most $12\Delta^2 - 36\Delta + 9$ vertices.

(3) Every graph with maximum degree $\Delta \geq 2$ admits an edge partition into $\lfloor \frac{\Delta+1}{2} \rfloor$ subgraphs H_i such that each connected component of H_i has at most $60\Delta - 63$ edges.

(4) For every integer n, there is a planar graph of maximum degree six such that in every vertex partition and every edge partition $\{G_1, G_2\}$, one of G_1, G_2 must have a connected component with at least n vertices, and there is a planar graph such that in every vertex partition $\{G_1, G_2, G_3\}$, one of G_1, G_2, G_3 must have a connected component with at least n vertices.

Reviewer: Van Bang Le (Rostock)

MSC:

05C15 Coloring of graphs and hypergraphs
05C70 Edge subsets with special properties (factorization, matching, partitioning, covering and packing, etc.)

Keywords:

tree-width; vertex partition; edge partition; graph coloring

Full Text: DOI

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