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The chromatic number of a graph of girth 5 on a fixed surface. (English) Zbl 1020.05030

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In this interesting paper it is proved that, for every surface S and every natural number k, there exists a natural number $f(S,k)$ such that the following holds: If G is a graph of girth 5 on S, and H is a 3-colored subgraph with at most k vertices, then either the coloring of H can be extended to a 3-coloring of G, or else there is a small obstruction containing H, that is, a subgraph H' with at most $f(S,k)$ vertices such that the coloring of H cannot be extended to a 3-coloring of H'. In particular, there are only finitely many 4-color-critical graphs of girth 5 on S, as a 4-color-critical graph of girth 5 on S has at most $f(S,1)$ vertices. It follows that, if G is a graph of girth 5 on S, and all noncontractible cycles in G have length greater than $f(S,1)$, then G is 3-colorable. The result is best possible in the sense that there are infinitely many 4-color-critical graphs of girth 4 on S, except when S is the sphere. As a consequence, it is deduced that the chromatic number of graphs of girth 5 on S can be found in polynomial time.

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