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Group actions on spaces of rational functions. (English) Zbl 1026.55011

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This paper continues and extends previous work of the second-named author and others on the homotopy type of certain kinds of function spaces. Let Hol_d denote the space of all holomorphic maps of degree $d \geq 0$ from the Riemann sphere S^2 to itself. Then for each d there is a corresponding evaluation fibration sequence $\mathrm{Hol}_d^* \to \mathrm{Hol}_d \to S^2$ with fibre Hol_d^* consisting of basepoint-preserving holomorphic maps. The function spaces Hol_d and Hol_d^* are of interest from a number of points of view and many results are known concerning the homotopy type of these function spaces. One connection with other areas of mathematics arises as follows: Denote the orbit space $\mathrm{Hol}_1 \backslash \mathrm{Hol}_d$ of the obvious action of Hol_1 on Hol_d by X_d. Then a theorem of Milgram says that, for $d \geq 1$, X_d is homeomorphic to the space of non-singular $d \times d$ Toeplitz matrices.

Here, the authors focus on the homotopy type of the universal covers $\widetilde{\mathrm{Hol}_d}$ and $\widetilde{\mathrm{Hol}_d^*}$. Their main results give homotopy equivalences as follows for $d \geq 1$: $\widetilde{\mathrm{Hol}_d} \simeq S^3 \times \widetilde{X}_d$ (Theorem 1.4), and $\widetilde{X}_d \simeq \widetilde{\mathrm{Hol}_d^*}$ (Theorem 1.5). From these results, the homotopy equivalence $\widetilde{\mathrm{Hol}_d} \simeq S^3 \times \widetilde{\mathrm{Hol}_d^*}$ is evident and the isomorphisms $\pi_k(\mathrm{Hol}_d) \cong \pi_k(S^3) \oplus \pi_{k+2}(S^2)$, for $2 \leq k < d$ may be obtained.

The last two consequences are also obtained in [*K. Yamaguchi*, Kyushu J. Math. 56, 381-387 (2002; Zbl 1041.55005)]. In the case $d = 2$, the homotopy types of Hol_2, $\widetilde{\mathrm{Hol}_2}$, and $\widetilde{\mathrm{Hol}_2^*}$ have been explicitly identified as homogeneous spaces in [*M. Guest, A. Kozlowski, M. Murayama*, and *K. Yamaguchi*, J. Math. Kyoto Univ. 35, 631-638 (1995; Zbl 0862.55011)]. This latter paper also contains computations of some homotopy groups $\pi_k(\mathrm{Hol}_d)$, which are obtained again in the paper under review. In these previous papers, the main tool – at least for the homotopy calculations – was the evaluation fibration sequence $\mathrm{Hol}_d^* \to \mathrm{Hol}_d \to S^2$ and its interplay with the evaluation fibration sequence $\mathrm{Map}_d^* \to \mathrm{Map}_d \to S^2$ obtained by considering continuous maps of degree d from S^2 to itself. In the paper under review, on the other hand, the results flow from a study of the action of Hol_1 on Hol_d. That this approach is fruitful is due in large part to the authors' skill in drawing on a wealth of facts and previous results in this area.

Reviewer: Gregory Lupton (Cleveland)

MSC:

55P15 Classification of homotopy type
55Q52 Homotopy groups of special spaces
55P10 Homotopy equivalences in algebraic topology
55P35 Loop spaces

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