Bergamasco, Adalberto P.; **Nunes, Wagner V. L.**; **Zani, Sérgio Luís**
Global properties of a class of overdetermined systems. (English) ⎹ Zbl 1034.32024 ⎸
J. Funct. Anal. 200, No. 1, 31-64 (2003).

The authors study a class of overdetermined systems of two complex linear differential equations on the three-dimensional real torus with one unknown function described as follows. Let $\varphi = a\,dt + b\,ds$ be a real analytic closed differential 1-form on the two-dimensional torus with periodic coordinates (t,s) and let Π be the product of the torus and the circle with coordinate x. Then $\psi = dx + i\varphi$, $i = \sqrt{-1}$, is a complex 1-form on Π. The operator of the system in question is formed by a pair of complex vector fields $v_1 = \frac{\partial}{\partial t} - ia\frac{\partial}{\partial x}$ and $v_2 = \frac{\partial}{\partial s} - ib\frac{\partial}{\partial x}$ which are orthogonal to ψ. In other words: the total differential $du = u_x dx + u_s ds + u_t dt$ of the unknown function is equal modulo of the constructed form ψ to $\chi = f\,dt + g\,ds$ determining the right side of the system.

Problems of the existence of solutions for such systems and their smoothness are investigated in detail. In particular, it is proved that under natural requirements on compatibility and periodicity a necessary and sufficient condition of the global solvability of the system is in fact its local solvability. In a similar case the latter condition was considered by *L. Nirenberg* and *F. Trèves* [Commun. Pure Appl. Math. 16, 331–351 (1963; Zbl 0117.06104)]; in the context of the paper under review one can formulate it in such a way: the local primitives of the differential form φ determine maps which are open at all the regular points of the zero-set defined by the coefficients of the 1-form. If the local condition fails then the system turns out to be non hypoelliptic. In order to prove this claim the authors construct a non-smooth function which is transformed into smooth ones by the vector fields v_1 and v_2.

Reviewer: Aleksandr G. Aleksandrov (Moskva)

MSC:

32W50	Other partial differential equations of complex analysis in several variables
35N10	Overdetermined systems of PDEs with variable coefficients
35H10	Hypoelliptic equations

Cited in **1** Review
Cited in **14** Documents

Keywords:

global solvability; hypoelliptic systems; analytic singularities; level sets; pseudoperiodic functions; commensurable periods; locally integrable structures; Euler graph; stationary phase; overdetermined systems; three-dimensional real torus

Full Text: DOI

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