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**Is network traffic approximated by stable Lévy motion or fractional Brownian motion?**  
(English) [Zbl 1021.60076](#)

*Ann. Appl. Probab.* 12, No. 1, 23-68 (2002).

From the authors' abstract: Cumulative broadband network traffic is often thought to be well modeled by fractional Brownian motion (FBM). However, some traffic measurements do not show an agreement with the Gaussian marginal distribution assumption. The authors show that if connection rates are modest relative to heavy tailed connection length distribution tails, then stable Lévy motion is a sensible approximation to cumulative traffic over a time period. If connection rates are large relative to heavy tailed connection length distribution tails, then FBM is the appropriate approximation. The results are framed as limit theorems for a sequence of cumulative input processes whose connection rates are varying in such a way as to remove or induce long range dependence.

Reviewer: [V.Schmidt \(Ulm\)](#)

**MSC:**

[60K25](#) Queueing theory (aspects of probability theory)  
[60F05](#) Central limit and other weak theorems

Cited in **1** Review  
Cited in **93** Documents

**Keywords:**

[heavy tails](#); [regular variation](#); [self-similarity](#); [stable Lévy motion](#); [fractional Brownian motion](#); [workload](#)

**Full Text:** [DOI](#)

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