

**Schwede, Stefan; Shipley, Brooke**

**Stable model categories are categories of modules.** (English) Zbl 1013.55005  
Topology 42, No. 1, 103-153 (2003).

A stable model category is a pointed closed model category for which the suspension functor is invertible up to homotopy, that is, stable model categories are those model categories which share the main formal property of spectra. The authors generalize this framework to several known results:

A classical theorem of *P. Gabriel* [Bull. Soc. Math. Fr. 90, 323-448 (1962; [Zbl 0201.35602](#))] characterizes categories of modules as the cocomplete abelian categories with a single small projective generator. In this paper it is proved that stable model categories with a single compact generator are equivalent to modules over a ring spectrum, and stable model categories with a set of generators are characterized as modules over a ring spectrum with several objects. This generalization of Gabriel's theorem develops a Morita theory for stable model categories.

The classical Morita theory describes equivalences between module categories in terms of bimodules [see *J. Adámek* and *J. Rosický*, *Locally Presentable and accessible Categories* (London Mathematical Society Lecture Notes Series, Vol. 189, Cambridge University Press, Cambridge) (1994; [Zbl 0795.18007](#))]. In this paper it is shown how equivalences between module categories over ring spectra can be realized by smashing with a pair of bimodules.

*J. Rickard* answered the question of when various derived module categories are equivalent as triangulated categories [J. Lond. Math. Soc., II. Ser. 39, No. 3, 436-456 (1989; [Zbl 0642.16034](#)); J. Lond. Math. Soc., II. Ser. 43, No. 1, 37-48 (1991; [Zbl 0683.16030](#))]. In this paper, stable model categories which represent the derived category of a ring are characterized.

The authors emphasize the conceptual similarities between the paper under review and the work of *B. Keller* [Ann. Sci. Éc. Norm. Supér., IV. Sér. 27, No. 1, 63-102 (1994; [Zbl 0799.18007](#))], but where Keller considers categories whose hom-objects are chain complexes of abelian groups they consider categories whose hom-objects are spectra (spectra are the homotopy-theoretical generalization of abelian groups). On the other hand, Keller does not use the language of model categories.

Reviewer: [Francisco J. Diaz \(La Laguna\)](#)

**MSC:**

- [55U35](#) Abstract and axiomatic homotopy theory in algebraic topology
- [55P42](#) Stable homotopy theory, spectra
- [18G55](#) Nonabelian homotopical algebra (MSC2010)
- [18E30](#) Derived categories, triangulated categories (MSC2010)

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