

**Konyagin, Sergei; Pappalardi, Francesco**

**Enumerating permutation polynomials over finite fields by degree.** (English) Zbl 1029.11067  
Finite Fields Appl. 8, No. 4, 548-553 (2002).

Every permutation on the elements of  $\mathbb{F}_q$  ( $q > 2$ ) is uniquely represented by a polynomial over  $\mathbb{F}_q$  of degree  $\leq q - 2$ . The authors deal with the problem of enumerating such permutation polynomials having degree  $< q - 2$ . This is equivalent to counting the permutations  $\sigma$  of  $\mathbb{F}_q$  for which  $\sum_{c \in \mathbb{F}_q} c\sigma(c) = 0$ .

Let  $N$  denote the number of permutations of  $\mathbb{F}_q$  satisfying this condition. By inclusion exclusion,

$$N = \sum_{S \subseteq \mathbb{F}_q} (-1)^{q-|S|} n_S$$

where  $n_S$  is the number of mappings  $f: \mathbb{F}_q \rightarrow S$  with  $\sum_{c \in S} cf(c) = 0$ . Using an expression for  $n_S$  in terms of exponential sums, the authors then show that

$$|N - (q - 1)!| \leq \sqrt{2e/\pi} q^{q/2}.$$

A similar estimate for a prime  $q$  was determined, using a different method, by *P. Das* [Finite Fields Appl. 8, 478-490 (2002; [Zbl 1029.11066](#))].

Reviewer: [Astrid Reifegerste \(Hannover\)](#)

**MSC:**

[11T06](#) Polynomials over finite fields

Cited in **2** Reviews  
Cited in **9** Documents

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**References:**

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