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Convex-monotone functions and generalized solution of parabolic Monge–Ampère equation.

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J. Differ. Equations 186, No. 2, 558-571 (2002).

The authors extend and improve their previous results [J. Partial Differ. Equations 14, 149-162 (2001; Zbl 0990.35034)] on the existence of generalized solutions of the first initial-boundary value problem for the parabolic Monge-Ampère equation

$$\begin{aligned} -u_t \det D^2 u = f(x,t) \quad &\text{in} \quad Q = \Omega \times (0,T], \\ u = \varphi(x,t) \quad &\text{on} \quad \partial_p Q. \end{aligned} \tag{*}$$

The paper contains two main results. The first is the Hölder continuity in t of u if $\varphi(t, x_0)$ is Hölder continuous in t for each $x_0 \in \partial\Omega$. The second is a geometric characterization of the convex-monotone solution U of (*) with $f \equiv 0$ as follows:

$$U(x,t) = \sup\{l(x) : l \text{ is affine and } l(x) \leq \varphi(x,0) \text{ in } \Omega,$$

$$l(x) \leq \varphi(x,t) \text{ on } \partial\Omega\}, \qquad (x,t) \in \bar{Q}.$$

Using these results the authors establish the existence of generalized solutions of (*) under somewhat weaker assumptions than required in previous work.

Reviewer: John Urbas (Bonn)

MSC:

35K55 Nonlinear parabolic equations

35D05 Existence of generalized solutions of PDE (MSC2000)

35D10 Regularity of generalized solutions of PDE (MSC2000)

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