Liu, James H.

A remark on the mild solutions of non-local evolution equations. (English) Zbl 1015.37045

Semigroup Forum 66, No. 1, 63-67 (2003).

This paper is devoted to study the nonlocal evaluation equation

$$\begin{cases} u'(t) = Au(t) + f(t, u(t)) \\ u(0) + g(u) = u_0, \end{cases} \qquad 0 \le t \le T, \tag{1}$$

where $g : C([0,T], X) \to X$ is a continuous function, and X is a general Banach space. To study (1), a very common approach is to define a map $F : C([0,T], X) \to C([0,T], X)$ by

$$F(u)(t) = T(t)[u_0 - g(u)] + \int_0^t T(t-s) f(s, u(s)) ds, \qquad 0 \le t \le T, \tag{2}$$

and prove that F has a fixed point, which is called a mild solution of (1). Here $T(\cdot)$ is the corresponding semigroup generated by (1).

The author addresses to the following question: Can the map F defined by (2) be a compact operator? In the case $u_0 = 0$, $f(\cdot, \cdot) = 0$, this question becomes: Can the map defined by

$$[T(\cdot)g](u) := T(\cdot)(g(u)), \quad u \in C([0,T], X)$$

be compact? The author shows that, the answer is no in general.

Reviewer: Messoud Efendiev (Berlin)

MSC:

37L05 General theory of infinite-dimensional dissipative dynamical systems, nonlinear semigroups, evolution equations

34G20 Nonlinear differential equations in abstract spaces

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