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Fixed point theorems for a broad class of multimaps. (English) Zbl 1029.54049

Nonlinear Anal., Theory Methods Appl., Ser. A, Theory Methods 52, No. 3, 961-969 (2003).

We say that $\varphi : \mathbb{R}^+ \to [0,1)$ has property A if there exists an $M \in (0,\infty)$ such that φ is monotonically increasing on $[M,\infty)$, $\int_M^\infty (1-\varphi(t))dt = +\infty$ and $\sup\{\varphi(t) : r \leq t \leq M\} < 1$, $\forall r \in (0,M)$, φ has property B if $\sup\{\varphi(t) : r \leq t < \infty\} < 1$, $\forall r \in (0,\infty)$. Let (X,d) be a metric space, $N(X)$ the collection of all nonempty subsets of X and $F : X \to N(X)$. We say that φ has property C w.r.t. (F,x_0) if it is monotonically increasing in $\mathbb{R}^+$ and $\int_0^\infty (1-\varphi(t))dt > d(x_0,Fx_0)$.

The main result of this paper is the following theorem: Let $F : X \to C(X)$, where $C(X)$ is the collection of all nonempty closed subsets of X. Suppose that (X,d) is F-orbitally complete, the function f defined on X as $f(x) = d(x,Fx)$ is F-orbitally semicontinuous at any cluster point of any orbit of F w.r.t. x_0, the function φ has property A or B or property C w.r.t. (F,x_0) and that $d(y,Fy) \leq \varphi(d(x_0,x))d(x,y)$ whenever $x \in 0(F,x_0)$, $y \in Fx$ and $x \notin Fx$. Then there exists an orbit $\{x_n\}_0^\infty$ of F w.r.t. x_0 which converges to a fixed point of F.

This theorem is a proper generalization of Theorem 2.1 of [*C.-K. Zhong*, *J. Zhu* and *P.-H. Zhao*, Proc. Am. Math. Soc. 128, No. 8, 2439-2444 (2000; Zbl 0948.47058)].

Reviewer: V.Popa (Bacau)

MSC:

54H25	Fixed-point and coincidence theorems (topological aspects)	Cited in **7** Documents
47H10	Fixed-point theorems	
54C60	Set-valued maps in general topology	
54H10	Topological representations of algebraic systems	

Keywords:

multimap; fixed point; orbital completeness; orbital lower semicontinuity

Full Text: DOI

References:

[1] Hicks, T.L.; Rhoades, B.E., A Banach type fixed point theorem, Math. japon., 24, 327-330, (1979) · Zbl 0432.47036

[2] Nadler, S.B., Multivalued contraction mappings, Pacific J. math., 30, 475-487, (1969)

[3] Ray, W.O.; Walker, A.M., Mapping theorems for Gâteaux differentiable and accretive operators, Nonlinear anal., 6, 5, 423-433, (1992) · Zbl 0488.47031

[4] Zhong, C.K., A generalization of Ekeland's variational principle and applications to the study of the relation between the weak P.S. condition and coercivity, Nonlinear anal., 29, 1421-1431, (1997) · Zbl 0912.49021

[5] Zhong, C.K.; Zhu, J.; Zhao, P.H., An extension of multi-valued contraction mappings and fixed points, Proc. amer. math. soc., 128, 2439-2444, (2000) · Zbl 0948.47058