Basarab, Şerban A.
Partially commutative Artin-Coxeter groups and their arboreal structure. (English)
Zbl 1050.20028
J. Pure Appl. Algebra 176, No. 1, 1-25 (2002).

Summary: Given a group G and a set $S \subseteq G$ of generators, set $S^{-1} = \{s^{-1} \mid s \in G\}$ and $\widetilde{S} = S \cup S^{-1}$. For $g \in G$, let $l(g)$ denote the minimum length of any expression $g = s_1 \cdots s_d$ with $s_i \in \widetilde{S}$. For $g, h \in G$, set $g \subset h$ if $l(g) + l(g^{-1}h) = l(h)$.

The paper is devoted to the study of the pairs (G, S) for which $1 \notin S$, $S \cap S^{-1} = S_1 := \{s \in S \mid s^2 = 1\}$, and the partial order $\subset$ satisfies the following conditions: (i) $(G, \subset)$ is a semilattice; denote by $g \cap h$ the greatest lower bound w.r.t. the order $\subset$ for any pair (g, h) of elements of G, (ii) $g^{-1}(g \cap h) \subset g^{-1}h$ for all $g, h \in G$, and (iii) $gh = hg$ is the least upper bound $g \cup h$ w.r.t. $\subset$ for the pair (g, h) whenever $g \cap h = 1$ and there exists $u \in G$ such that $g \subset u$ and $h \subset u$.

It is shown that the pairs above are exactly those for which G admits the presentation $G = \langle S;\ s^2 = 1$ for $s \in S_1$, and $sts^{-1}t^{-1} = 1$ for those $s, t \in S$, $s \neq t$, for which the commuting relation $st = ts$ holds in $G\rangle$. Call the groups defined by such presentations 'partially commutative Artin-Coxeter groups'.

The pairs (G, S) above satisfy a 'deletion condition' (D) analogous to the well-known deletion condition for Coxeter groups. It is shown that the pairs (G, S) satisfying (D) have solvable word problem, as is the case with usual Coxeter groups.

Normal forms for elements in partially commutative Artin-Coxeter groups are also described.

MSC:

20F55	Reflection and Coxeter groups (group-theoretic aspects)
20F05	Generators, relations, and presentations of groups
20F10	Word problems, other decision problems, connections with logic and automata (group-theoretic aspects)
20F36	Braid groups; Artin groups
05C25	Graphs and abstract algebra (groups, rings, fields, etc.)

Keywords:

partially commutative Artin-Coxeter groups; presentations; word problem; normal forms

Full Text: DOI

References:

[1]   R.C. Alperin (Ed.), Arboreal Group Theory, Mathematical Sciences Research Publications, Vol. 19, Springer, Berlin, 1991.

[2]   Alperin, R.C.; Bass, H., Length functions of group actions on \textit{λ}-trees, (), 265-378 · Zbl 0978.20500

[3]   Basarab, Ş.A., On a problem raised by Alperin and bass, (), 35-68 · Zbl 0808.20030

[4]   Basarab, Ş.A., On a problem raised by Alperin and bass I: group actions on groupoids, J. pure appl. algebra, 73, 1-12, (1991) · Zbl 0741.20017

[5]   Ş.A. Basarab, The dual of the category of trees, Preprint Series of the Institute of Mathematics of the Romanian Academy No. 7, 1992, 21pp.

[6]   Ş.A. Basarab, On a problem raised by Alperin and Bass II: metric and order theoretic aspects, Preprint Series of the Institute of Mathematics of the Romanian Academy No. 10, 1992, 30pp.

[7]   Basarab, Ş.A., Directions and foldings on generalized trees, Fund. inform., 30, 2, 125-149, (1997) · Zbl 0888.68091

[8]   Ş.A. Basarab, Partially commutative Artin-Coxeter groups and their arboreal structure, I, II, Preprint Series of the Institute of Mathematics of the Romanian Academy No. 5, 1997, 20pp; 7, 1997, 26pp.

[9]   Basarab, Ş.A., On discrete hyperbolic arboreal groups, Commun. algebra, 26, 9, 2837-2865, (1998) · Zbl 0921.20044

[10]   Ş.A. Basarab, The arithmetic-arboreal residue structure of a Prüfer domain I, in: Valuation Theory and Its Applications, Vol. I, in: Fields Institute Communications, Vol. 32, 2001, to appear.

[11] N. Bourbaki, Groupes et Algébres de Lie, Mason, Paris 1981 (Chapitres 4-6). · Zbl 0483.22001

[12] Brieskorn, E.; Saito, K., Artin – gruppen und coxeter – gruppen, Invent. math., 17, 245-271, (1972) · Zbl 0243.20037

[13] Brown, K.S., Buildings, (1989), Springer Berlin

[14] Cartier, P.; Foata, D., Problèmes combinatoires de commutation et réarrangements, Lecture notes in mathematics, Vol. 85, (1969), Springer Berlin · Zbl 0186.30101

[15] Deligne, P., LES immeubles des groupes de tresses généralisés, Invent. math., 17, 273-302, (1972) · Zbl 0238.20034

[16] Epstein, D., Word processing in groups, (1992), Jones and Bartlett Publishers Boston, MA · Zbl 0764.20017

[17] Morgan, J.; Shalen, P., Valuations, trees and degenerations of hyperbolic structures I, Ann. math., 120, 401-476, (1984) · Zbl 0583.57005

[18] M. Roller, Poc sets, median algebras and group actions. An extended study of Dunwoody's construction and Sageev's theorem, Southampton Preprint Archive, 1998, http://www.maths.soton.ac.uk/pure/reprints.htm.

[19] Serre, J.P., Trees, (1986), Springer Berlin

[20] Sholander, M., Trees, lattices, order, and betweenness, Proc. amer. math. soc., 3, 369-381, (1952)

[21] J. Tits, Le probléme des mots dans les groupes de Coxeter, Symposia Mathematica (INDAM, Rome, 1976/68), Vol. 1, Academic Press, New York, 1969, pp. 175-185.