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Signature asymptotique d'un champ de vecteurs en dimension 3. (Asymptotic signature of a vector field in dimension 3). (French. English summary) Zbl 1010.37010

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Summary: Consider a volume preserving vector field defined in some compact domain of 3-space and tangent to its boundary. A long piece of orbit can be made into a knot by connecting its endpoints by some arc whose length is less than the diameter of the domain. In this paper, we study the behaviour of the signatures of these knots as the lengths of the pieces of orbits go to infinity. We relate this "asymptotic signature" to the "asymptotic Hopf invariant" that has been studied by *V. I. Arnol'd* [Sel. Math. Soc. 5, 327-345 (1986; Zbl 0623.57016)].

MSC:

37C10 Dynamics induced by flows and semiflows
37C50 Approximate trajectories (pseudotrajectories, shadowing, etc.) in smooth dynamics
57R25 Vector fields, frame fields in differential topology
57M25 Knots and links in the 3-sphere (MSC2010)

Keywords:

asymptotic Hopf invariant; volume preserving vector field

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