**Kapovich, Ilya**; **Benakli, Nadia**
**Boundaries of hyperbolic groups.** (English) ⎡Zbl 1044.20028⎤
Cleary, Sean (ed.) et al., Combinatorial and geometric group theory. Proceedings of the AMS special session on combinatorial group theory, New York, NY, USA, November 4–5, 2000 and the AMS special session on computational group theory, Hoboken, NJ, USA, April 28–29, 2001. Providence, RI: American Mathematical Society (AMS) (ISBN 0-8218-2822-3/ pbk). Contemp. Math. 296, 39-93 (2002).

To every Gromov-hyperbolic space $X$ (in particular to every word-hyperbolic group) one can associate the so-called "space at infinity" or boundary $\partial X$. The boundary has a rich structure (topological, dynamical, metric, quasiconformal, measure-theoretic, and algebraic) with deep and important theory and numerous applications. The authors aim to give a broad overview of known results, methods and ideas involved in studying and using the hyperbolic boundary. The focus of the article is more on the boundaries of hyperbolic groups and to a lesser extent on the subject of boundaries of Gromov-hyperbolic spaces. The authors not provide precise proofs but rather give an informal non-technical discussion of the main arguments. However they start with a careful presentation of the basic definitions and concepts.

Moreover, in the article are discussed the following topics: 1. quasiconformal and quasi-Möbius structures on the boundary; 2. basic aspects of the topological dynamics of the action by a word-hyperbolic group on its boundary; 3. hyperbolic groups as convergence group actions; 4. cohomological properties of a hyperbolic group and its boundary; 5. analysis of the relationship between local connectivity of the boundary and splittings of hyperbolic groups over finite and virtually cyclic subgroups; 6. overview of hyperbolic groups with zero- and one-dimensional boundary; 7. work of J. Cannon on groups with boundary homeomorphic to the two-sphere; 8. random walks and representing hyperbolic boundary as the Poisson boundary and the Martin boundary; 9. connections with $C^*$-algebras, Gromov's conformal boundary and the Novikov conjecture; 10. the use of the boundary considerations for investigation of the subgroup structure of hyperbolic groups; 11. a brief introduction to the notion of a geodesic flow; 12. Pansu-Gromov conformal dimension of the boundary; 13. Sullivan-Patterson measures; 14. symbolic dynamics and viewing a hyperbolic group and its boundary as an abstract dynamical system. Finally, various miscellaneous topics are presented. – We have to mention that the article is well written and the reader can find a lot of very fascinating parts.

For the entire collection see [Zbl 0990.00044].

Reviewer: Andrzej Szczepański (Gdańsk)

**MSC:**

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|---|---|---|
| 20F67 | Hyperbolic groups and nonpositively curved groups | Cited in **58** Documents |
| 20F65 | Geometric group theory | |
| 20F69 | Asymptotic properties of groups | |
| 57M07 | Topological methods in group theory | |
| 53C23 | Global geometric and topological methods (à la Gromov); differential geometric analysis on metric spaces | |
| 37B10 | Symbolic dynamics | |

**Keywords:**

hyperbolic groups; hyperbolic boundary spaces at infinity

**Full Text:** arXiv